Decisions and Prospective Rationality

Richard Bradley

January 2012

Agents

1. What does decision making require?

Agents

1. What does decision making require?
   - Representations of features of the environment

Agents

1. What does decision making require?
   - Representations of features of the environment
   - Ability to infer absence or presence of features i.e. form beliefs

Agents

1. What does decision making require?
   - Representations of features of the environment
   - Ability to infer absence or presence of features i.e. form beliefs
   - Ability to evaluate features

Agents

1. What does decision making require?
   - Representations of features of the environment
   - Ability to infer absence or presence of features i.e. form beliefs
   - Ability to evaluate features
2. Reactive versus prospective agency

Agents

1. What does decision making require?
   - Representations of features of the environment
   - Ability to infer absence or presence of features i.e. form beliefs
   - Ability to evaluate features
2. Reactive versus prospective agency
3. Learning and retrospective evaluation

Agents

1. What does decision making require?
   - Representations of features of the environment
   - Ability to infer absence or presence of features i.e. form beliefs
   - Ability to evaluate features
2. Reactive versus prospective agency
3. Learning and retrospective evaluation
4. **Aim**: To formulate a precise theory of prospective rationality

Structure of Course

1. Introduction

Structure of Course

1. Introduction
   - Decision making and Uncertainty

Structure of Course

1. Introduction
   - Decision making and Uncertainty
   - The Logic of Prospects

Structure of Course

1. Introduction
   - Decision making and Uncertainty
   - The Logic of Prospects
2. Rational Partial Attitudes

Structure of Course

1. Introduction
   - Decision making and Uncertainty
   - The Logic of Prospects
2. Rational Partial Attitudes
   - Degrees of belief and desire

Structure of Course

1. Introduction
   - Decision making and Uncertainty
   - The Logic of Prospects
2. Rational Partial Attitudes
   - Degrees of belief and desire
   - Conditional belief and desire

Structure of Course

1. Introduction
   - Decision making and Uncertainty
   - The Logic of Prospects
2. Rational Partial Attitudes
   - Degrees of belief and desire
   - Conditional belief and desire
   - Attitudes to conditional prospects

Structure of Course

1. Introduction
   - Decision making and Uncertainty
   - The Logic of Prospects
2. Rational Partial Attitudes
   - Degrees of belief and desire
   - Conditional belief and desire
   - Attitudes to conditional prospects
3. Multidimensional Possible Worlds models

Structure of Course

1. Introduction
   - Decision making and Uncertainty
   - The Logic of Prospects
2. Rational Partial Attitudes
   - Degrees of belief and desire
   - Conditional belief and desire
   - Attitudes to conditional prospects
3. Multidimensional Possible Worlds models
   - Semantic models for conditionals

Structure of Course

1. Introduction
   - Decision making and Uncertainty
   - The Logic of Prospects
2. Rational Partial Attitudes
   - Degrees of belief and desire
   - Conditional belief and desire
   - Attitudes to conditional prospects
3. Multidimensional Possible Worlds models
   - Semantic models for conditionals
   - Counterfactual evaluation

Structure of Course

1. Introduction
   - Decision making and Uncertainty
   - The Logic of Prospects
2. Rational Partial Attitudes
   - Degrees of belief and desire
   - Conditional belief and desire
   - Attitudes to conditional prospects
3. Multidimensional Possible Worlds models
   - Semantic models for conditionals
   - Counterfactual evaluation
4. Action

Structure of Course

1. Introduction
   - Decision making and Uncertainty
   - The Logic of Prospects
2. Rational Partial Attitudes
   - Degrees of belief and desire
   - Conditional belief and desire
   - Attitudes to conditional prospects
3. Multidimensional Possible Worlds models
   - Semantic models for conditionals
   - Counterfactual evaluation
4. Action
   - The Ramsey-Jeffrey-Savage Synthesis

Structure of Course

1. Introduction
   - Decision making and Uncertainty
   - The Logic of Prospects
2. Rational Partial Attitudes
   - Degrees of belief and desire
   - Conditional belief and desire
   - Attitudes to conditional prospects
3. Multidimensional Possible Worlds models
   - Semantic models for conditionals
   - Counterfactual evaluation
4. Action
   - The Ramsey-Jeffrey-Savage Synthesis
   - Causal Decision Theory

Structure of Course

1. Introduction
   - Decision making and Uncertainty
   - The Logic of Prospects
2. Rational Partial Attitudes
   - Degrees of belief and desire
   - Conditional belief and desire
   - Attitudes to conditional prospects
3. Multidimensional Possible Worlds models
   - Semantic models for conditionals
   - Counterfactual evaluation
4. Action
   - The Ramsey-Jeffrey-Savage Synthesis
   - Causal Decision Theory
5. A Representation theorem (METEORITE talk)

Savage's Decision Theory

- Standard theory due to Leonard Savage (1954).

Savage's Decision Theory

- Standard theory due to Leonard Savage (1954).
  - Tripartite distinction between states (events), consequences, and acts

Savage's Decision Theory

- Standard theory due to Leonard Savage (1954).
  - Tripartite distinction between states (events), consequences, and acts
  - Uncertainty is subjective and confined to states

Savage's Decision Theory

- Standard theory due to Leonard Savage (1954).
  - Tripartite distinction between states (events), consequences, and acts
  - Uncertainty is subjective and confined to states
  - States are act independent;consequences are state independent

Savage's Decision Theory

- Standard theory due to Leonard Savage (1954).
  - Tripartite distinction between states (events), consequences, and acts
  - Uncertainty is subjective and confined to states
  - States are act independent;consequences are state independent
- **State-Consequence Matrix**

	States of the world		
Options	S_1	...	S_n
A_1	C_{11}	...	C_{1n}
...	...	...	...
A_m	C_{m1}	...	C_{mn}

Savage's Decision Theory

- Standard theory due to Leonard Savage (1954).
  - Tripartite distinction between states (events), consequences, and acts
  - Uncertainty is subjective and confined to states
  - States are act independent;consequences are state independent
- **State-Consequence Matrix**

	States of the world		
Options	S_1	...	S_n
A_1	C_{11}	...	C_{1n}
...	...	...	...
A_m	C_{m1}	...	C_{mn}

- Representation theorem for state-independent subjective expected utility:

$$SEU(A_i) = \sum_j \Pr(S_i).U(C_{ij})$$

Types of Uncertainty

- Tripartite classification of uncertainty:

Types of Uncertainty

- Tripartite classification of uncertainty:
  1. **Grade**: Normal-Severe

Types of Uncertainty

- Tripartite classification of uncertainty:
  1. **Grade**: Normal-Severe
  2. **Type**: Empirical-Ethical

Types of Uncertainty

- Tripartite classification of uncertainty:
  1. **Grade**: Normal-Severe
  2. **Type**: Empirical-Ethical
  3. **Mode**: Factual-Counterfactual

Types of Uncertainty

- Tripartite classification of uncertainty:
  1. **Grade**: Normal-Severe
  2. **Type**: Empirical-Ethical
  3. **Mode**: Factual-Counterfactual
- Standard theory only recognises state uncertainty: normal, empirical, factual

Types of Uncertainty

- Tripartite classification of uncertainty:
  1. **Grade**: Normal-Severe
  2. **Type**: Empirical-Ethical
  3. **Mode**: Factual-Counterfactual
- Standard theory only recognises state uncertainty: normal, empirical, factual
- Reasoning typically involves both:

Types of Uncertainty

- Tripartite classification of uncertainty:
  1. **Grade**: Normal-Severe
  2. **Type**: Empirical-Ethical
  3. **Mode**: Factual-Counterfactual
- Standard theory only recognises state uncertainty: normal, empirical, factual
- Reasoning typically involves both:
  1. Ethical uncertainty

Types of Uncertainty

- Tripartite classification of uncertainty:
  1. **Grade**: Normal-Severe
  2. **Type**: Empirical-Ethical
  3. **Mode**: Factual-Counterfactual
- Standard theory only recognises state uncertainty: normal, empirical, factual
- Reasoning typically involves both:
  1. Ethical uncertainty
  2. Option uncertainty i.e. uncertainty about what would happen if an action were performed

Prospects

- Prospects = objects of attitudes

Prospects

- Prospects = objects of attitudes
  - States, consequences, actions

Prospects

- Prospects = objects of attitudes
  - States, consequences, actions
  - Conditional prospects

Prospects

- Prospects = objects of attitudes
  - States, consequences, actions
  - Conditional prospects
- Role of language

Prospects

- Prospects = objects of attitudes
  - States, consequences, actions
  - Conditional prospects
- Role of language
  - Semantic framework (set-theoretic versus logical)

Prospects

- Prospects = objects of attitudes
  - States, consequences, actions
  - Conditional prospects
- Role of language
  - Semantic framework (set-theoretic versus logical)
  - Logic of prospects

Notation

1. Prospects

Notation

1. Prospects
   - Arbitrary: $\alpha, \beta, \gamma, ...$

Notation

1. Prospects
   - Arbitrary: $\alpha, \beta, \gamma, ...$
   - Factuals: $A, B, C, ...$

Notation

1. Prospects
   - Arbitrary: $\alpha, \beta, \gamma, ...$
   - Factuals: $A, B, C, ...$
   - Impossible prospect $\perp$

Notation

1. Prospects
   - Arbitrary: $\alpha, \beta, \gamma, ...$
   - Factuals: $A, B, C, ...$
   - Impossible prospect $\bot$
   - Neccessary prospect $\top$

Notation

1. Prospects
   - Arbitrary: $\alpha, \beta, \gamma, ...$
   - Factuals: $A, B, C, ...$
   - Impossible prospect $\bot$
   - Neccessary prospect $\top$
2. Operations on prospects:

Notation

1. Prospects
   - Arbitrary: $\alpha, \beta, \gamma, ...$
   - Factuals: $A, B, C, ...$
   - Impossible prospect $\bot$
   - Neccessary prospect $\top$
2. Operations on prospects:
   - Conjunction: $\alpha \wedge \beta$ (or just $\alpha\beta$)

Notation

1. Prospects
   - Arbitrary: $\alpha, \beta, \gamma, ...$
   - Factuals: $A, B, C, ...$
   - Impossible prospect $\bot$
   - Neccessary prospect $\top$
2. Operations on prospects:
   - Conjunction: $\alpha \wedge \beta$ (or just $\alpha\beta$)
   - Disjunction: $\alpha \vee \beta$

Notation

1. Prospects
   - Arbitrary: $\alpha, \beta, \gamma, ...$
   - Factuals: $A, B, C, ...$
   - Impossible prospect $\bot$
   - Neccessary prospect $\top$
2. Operations on prospects:
   - Conjunction: $\alpha \wedge \beta$ (or just $\alpha\beta$)
   - Disjunction: $\alpha \vee \beta$
   - Negation: $\neg\alpha$

Notation

1. Prospects
   - Arbitrary: $\alpha, \beta, \gamma, ...$
   - Factuals: $A, B, C, ...$
   - Impossible prospect $\bot$
   - Neccessary prospect $\top$
2. Operations on prospects:
   - Conjunction: $\alpha \wedge \beta$ (or just $\alpha\beta$)
   - Disjunction: $\alpha \vee \beta$
   - Negation: $\neg\alpha$
   - Conditional: $\alpha \rightarrow \beta$

Notation

1. Prospects
   - Arbitrary: $\alpha, \beta, \gamma, ...$
   - Factuals: $A, B, C, ...$
   - Impossible prospect $\bot$
   - Neccessary prospect $\top$
2. Operations on prospects:
   - Conjunction: $\alpha \wedge \beta$ (or just $\alpha\beta$)
   - Disjunction: $\alpha \vee \beta$
   - Negation: $\neg\alpha$
   - Conditional: $\alpha \rightarrow \beta$
3. Preference relations on prospects: $\succsim, \succ, \approx$

Attitudes

.

- Attitudes = psyschological states that have both content and force/grade

Attitudes

.

- Attitudes = psyschological states that have both content and force/grade
- Attitudes are relations on prospects

Attitudes

.

- Attitudes = psyschological states that have both content and force/grade
- Attitudes are relations on prospects
  - Type of attitudes characterised by properties

Attitudes

.

- Attitudes = psyschological states that have both content and force/grade
- Attitudes are relations on prospects
  - Type of attitudes characterised by properties
  - Numerically representable under certain conditions

Attitudes

.

- Attitudes = psyschological states that have both content and force/grade
- Attitudes are relations on prospects
  - Type of attitudes characterised by properties
  - Numerically representable under certain conditions
- Properties of relations

Attitudes

.

- Attitudes = psyschological states that have both content and force/grade
- Attitudes are relations on prospects
  - Type of attitudes characterised by properties
  - Numerically representable under certain conditions
- Properties of relations
  1. *Completeness*: $\alpha \succeq \beta$ or $\beta \succeq \alpha$

Attitudes

.

- Attitudes = psyschological states that have both content and force/grade
- Attitudes are relations on prospects
  - Type of attitudes characterised by properties
  - Numerically representable under certain conditions
- Properties of relations
  1. *Completeness*: $\alpha \succeq \beta$ or $\beta \succeq \alpha$
  2. *Transitivity*: $\alpha \succeq \beta$ and $\beta \succeq \gamma$ then $\alpha \succeq \gamma$

Attitudes

.

- Attitudes = psyschological states that have both content and force/grade
- Attitudes are relations on prospects
  - Type of attitudes characterised by properties
  - Numerically representable under certain conditions
- Properties of relations
  1. *Completeness*: $\alpha \succeq \beta$ or $\beta \succeq \alpha$
  2. *Transitivity*: $\alpha \succeq \beta$ and $\beta \succeq \gamma$ then $\alpha \succeq \gamma$
  3. *Monotonicity*: $\alpha \vdash \beta$ then $\beta \succeq \alpha$

Attitudes

.

- Attitudes = psyschological states that have both content and force/grade
- Attitudes are relations on prospects
  - Type of attitudes characterised by properties
  - Numerically representable under certain conditions
- Properties of relations
  1. *Completeness*: $\alpha \succeq \beta$ or $\beta \succeq \alpha$
  2. *Transitivity*: $\alpha \succeq \beta$ and $\beta \succeq \gamma$ then $\alpha \succeq \gamma$
  3. *Monotonicity*: $\alpha \vdash \beta$ then $\beta \succeq \alpha$
  4. *Quasi-additivity*: If $\alpha \perp \gamma$ and $\alpha \perp \gamma$ then $\alpha \vee \gamma \succeq \beta \vee \gamma \Leftrightarrow \alpha \succeq \beta$

Attitudes

.

- Attitudes = psyschological states that have both content and force/grade
- Attitudes are relations on prospects
  - Type of attitudes characterised by properties
  - Numerically representable under certain conditions
- Properties of relations
  1. *Completeness*: $\alpha \succeq \beta$ or $\beta \succeq \alpha$
  2. *Transitivity*: $\alpha \succeq \beta$ and $\beta \succeq \gamma$ then $\alpha \succeq \gamma$
  3. *Monotonicity*: $\alpha \vdash \beta$ then $\beta \succeq \alpha$
  4. *Quasi-additivity*: If $\alpha \perp \gamma$ and $\alpha \perp \gamma$ then $\alpha \vee \gamma \succeq \beta \vee \gamma \Leftrightarrow \alpha \succeq \beta$
  5. *Betweenness*: If $\alpha \perp \beta$ then $\alpha \succeq (\alpha \vee \beta) \succeq \beta \Leftrightarrow \alpha \succeq \beta$

Characteristics of Attitudes

	Attitude Type		
Properties	*Cognitive*	*Pure Evaluative*	*Mixed Eval.*
Transitivity	X	X	X
Monotonicity	X		
Quasi-additivity	X		X
Betweenness		X	

Rational Belief

Thesis 1. (*Probabilism*) Rational degrees of belief are probabilities.

Definition (Probability)

A real-valued function P on a Boolean algebra of prospects is a probability iff:

P0 (*Non-negativity*) $P(\alpha) \geq 0$

P1 (*Normality*) $P(\top) = 1$

P2 (*Additivity*) If $\alpha\beta = \bot$, then:

$$P(\alpha \vee \beta) = P(\alpha) + P(\beta)$$

Rational Desire

Thesis 2. (*Desirabilism*) Rational degrees of desire are desirabilities

Definition (Desirability)

A real-valued function V on a Boolean algebra of prospects is a desirability iff:

V1 (*Normality*) $V(\top) = 0$

V2 (*Averaging*) If $\alpha\beta = \bot$ and $P(\alpha \vee \beta) \neq 0$ then:

$$V(\alpha \vee \beta) = \frac{V(\alpha).P(\alpha) + V(\beta).P(\beta)}{P(\alpha \vee \beta)}$$

Conditional Attitudes

- Conditional attitude = attitude under supposition of some condition

Conditional Attitudes

- Conditional attitude = attitude under supposition of some condition
  - $\neq$ attitude we would have were the condition true

Conditional Attitudes

- Conditional attitude = attitude under supposition of some condition
  - $\neq$ attitude we would have were the condition true
  - $\neq$ attitude to a conditional

Conditional Attitudes

- Conditional attitude = attitude under supposition of some condition
  - $\neq$ attitude we would have were the condition true
  - $\neq$ attitude to a conditional
- Different ways of supposing a conditional true

Conditional Attitudes

- Conditional attitude = attitude under supposition of some condition
  - $\neq$ attitude we would have were the condition true
  - $\neq$ attitude to a conditional
- Different ways of supposing a conditional true
  - Matter-of-fact versus counterfactual supposition

Conditional Attitudes

- Conditional attitude = attitude under supposition of some condition
  - $\neq$ attitude we would have were the condition true
  - $\neq$ attitude to a conditional
- Different ways of supposing a conditional true
  - Matter-of-fact versus counterfactual supposition
  - Evidential versus interventional supposition

Conditional Belief

Thesis 3: (*Conditional Belief*) Rational conditional degrees of belief are suppositional probabilities

Definition (Suppositional Probability)

P^*1 (*Probability*) P^*_α is a probability function

P^*2 (*Certainty*) $P^*_\alpha(\alpha) = P^*_\alpha(\top)$

P^*3 (*Regularity*) $P^*_\alpha(\alpha\beta) \geq P^*(\alpha\beta)$

P^+4 (*Conjunction*) $(P^+_\alpha)^+_\beta = P^+_{\alpha\beta}$

Conditional Desire

Thesis 4: (*Conditional Desire*) Rational conditional degrees of desire are suppositional desirabilities

Definition (**Suppositional Probability**)

V^*1 (*Desirability*) V^*_α is a desirability function

V^*2 (*Certainty*) $V^*_\alpha(\alpha) = V^*_\alpha(\top)$

V^*3 (*Regularity*) $\frac{V^*_\alpha(\alpha\beta)}{V^*_\alpha(\neg(\alpha\beta))} \geq \frac{V(\alpha\beta)}{V(\neg(\alpha\beta))}$

V^+4 (*Conjunction*) $(V^+_\alpha)^+_\beta = V^+_{\alpha\beta}$

Evidential Supposition (Belief)

Definition (Conditional Probability)

If $P(\alpha) \neq 0$, then:

$$P(\beta|\alpha) = \frac{P(\beta\alpha)}{P(\alpha)}$$

Thesis 5: (*Conditional Evidential Belief*) Rational degrees of belief on the evidential supposition that A are conditional probabilities given A.

Evidential Supposition (Desire)

Definition (Conditional Desirability)

If $P(\alpha\beta) \neq 0$, then:

$$V(\beta|\alpha) = V(\alpha\beta) - V(\alpha)$$

Thesis 6: (*Conditional Evidential Desire*) Rational degrees of desire on the evidential supposition that A are conditional desirabilities given A.

"If two people are arguing 'If p will q?' and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q; so that in a sense 'If p, q' and 'If p, ¬q' are contradictories." (Ramsey, 1929, p.155)

Thesis 7: (*Ramsey Test for belief*) Rational degrees of belief in conditionals equal degrees of belief in their consequent on the supposition of their antecedent; i.e.:

$$P(\alpha \rightarrow \beta) = P^*_\alpha(\beta)$$

Thesis 8 (*Adams*): $P(A \mapsto B) = P^+_A(B) = P(B|A)$

Thesis 9 (*Ramsey Test for Desire*) Rational degrees of desire in conditionals are proportional to the conditional desirabilities of their consequents on the supposition of their antecedent. Formally, and more exactly, for every prospect α there exists a real number $k_\alpha > 0$, such that for all prospects β:

$$V(\alpha \rightarrow \beta) = k_\alpha . V^*_\alpha(\beta)$$

Thesis 10: (*Bradley*) $V(A \mapsto B) = V^+_A(B).P(A) = V(B|A).P(A)$

Partitioning Conditionals

Thesis 11 (*Independence*) Orthogonal indicative conditionals are desirabilistically independent. Formally, if $\alpha(\alpha') = \bot$:

$$V(\alpha \mapsto \beta | \alpha' \mapsto \gamma) = V(\alpha \mapsto \beta)$$

Theorem

Independence implies that:

1. $V((\alpha \mapsto \beta)(\alpha' \mapsto \gamma)) = V(\alpha \mapsto \beta) + V(\alpha' \mapsto \gamma)$
2. $P((\alpha \mapsto \beta)(\alpha' \mapsto \gamma)) = P(\alpha \mapsto \beta).P(\alpha' \mapsto \gamma)$

Facing the Triviality results

- Are the 10 rationality theses consistent?

Facing the Triviality results

- Are the 10 rationality theses consistent?

Theorem (Triviality)

(Lewis) Suppose that Adams' Thesis is preserved under conditionalisation. Then for all prospects A and B:

$$P(\beta|\alpha) = P(\beta)$$

Facing the Triviality results

- Are the 10 rationality theses consistent?

Theorem (Triviality)

(Lewis) Suppose that Adams' Thesis is preserved under conditionalisation. Then for all prospects A and B:

$$P(\beta|\alpha) = P(\beta)$$

- *Diagnosis*: Ambiguity in the generalisation of Adams' Thesis:

Facing the Triviality results

- Are the 10 rationality theses consistent?

Theorem (Triviality)

(Lewis) Suppose that Adams' Thesis is preserved under conditionalisation. Then for all prospects A and B:

$$P(\beta|\alpha) = P(\beta)$$

- *Diagnosis*: Ambiguity in the generalisation of Adams' Thesis:
  1. $P(A \mapsto B|C) = \frac{P((A \mapsto B)C)}{P(C)}$

Facing the Triviality results

- Are the 10 rationality theses consistent?

Theorem (Triviality)

(Lewis) Suppose that Adams' Thesis is preserved under conditionalisation. Then for all prospects A and B:

$$P(\beta|\alpha) = P(\beta)$$

- *Diagnosis*: Ambiguity in the generalisation of Adams' Thesis:
  1. $P(A \mapsto B|C) = \frac{P((A \mapsto B)C)}{P(C)}$
  2. $P(A \mapsto B|C) = \frac{P(AB|C)}{P(A|C)} = P(B|AC)$

Facing the Triviality results

- Are the 10 rationality theses consistent?

Theorem (Triviality)

(Lewis) Suppose that Adams' Thesis is preserved under conditionalisation. Then for all prospects A and B:

$$P(\beta|\alpha) = P(\beta)$$

- *Diagnosis*: Ambiguity in the generalisation of Adams' Thesis:
  1. $P(A \mapsto B|C) = \frac{P((A \mapsto B)C)}{P(C)}$
  2. $P(A \mapsto B|C) = \frac{P(AB|C)}{P(A|C)} = P(B|AC)$
- 1. is required by definition of conditional probability, 2. by Conjunction axioms

Facing the Triviality results

- Are the 10 rationality theses consistent?

Theorem (Triviality)

(Lewis) Suppose that Adams' Thesis is preserved under conditionalisation. Then for all prospects A and B:

$$P(\beta|\alpha) = P(\beta)$$

- *Diagnosis*: Ambiguity in the generalisation of Adams' Thesis:
  1. $P(A \mapsto B|C) = \frac{P((A \mapsto B)C)}{P(C)}$
  2. $P(A \mapsto B|C) = \frac{P(AB|C)}{P(A|C)} = P(B|AC)$
- 1. is required by definition of conditional probability, 2. by Conjunction axioms
- *Solution*: Adams' thesis should be written:

$$P(\alpha \rightarrow \beta) = P^{+}_{\alpha}(\beta)$$