Lexicographic Beliefs

Part II: Respect of Preferences

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Introduction

- Cautious reasoning = not completely discarding any event, yet being able to consider some event much more likely, indeed infinitely more likely, than some other event
- Modelling tool: lexicographic beliefs
- A particular way of cautious reasoning is based on primary belief in rationality: restrictions concentrate only on the first lexicographic level
- However, it can also be plausible to impose conditions on deeper lexicographic levels!

Agenda

- Respecting the Opponent's Preferences
- Common Full Belief in (Caution & Respect of Preferences)
- Existence
- Towards an Algorithm
- Algorithm

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Taking the Opponent’s Preferences Seriously

Motivating Idea:

- If player i believes that his opponent j prefers some choice c_j to some other choice c_j', then he must deem c_j infinitely more likely than c_j'.

Motivating Example: Where to read my book?

Story

- *You* would like to go to a pub to read your book.
- *Barbara* is going to a pub as well, but *you* forgot to ask her to which one.
- Your only objective is to avoid *Barbara*, since *you* would like to read your book in silence.
- *Barbara* prefers *Pub A* to *Pub B*, and *Pub B* to *Pub C*.
- Question: Which pub should *you* go to?

Motivating Example: Where to read my book?

		Barbara		
		A	B	C
You	A	$0, 3$	$1, 2$	$1, 1$
	B	$1, 3$	$0, 2$	$1, 1$
	C	$1, 3$	$1, 2$	$0, 1$

Motivating Example: Where to read my book?

		Barbara		
		A	B	C
You	A	$0, 3$	$1, 2$	$1, 1$
	B	$1, 3$	$0, 2$	$1, 1$
	C	$1, 3$	$1, 2$	$0, 1$

- **Type Spaces:** $T_{you} = \{t_y\}$ and $T_{Barbara} = \{t_B\}$
- **Beliefs for** You: $b^{lex}_{you}(t_y) = ((A, t_B); (C, t_B); (B, t_B))$
- **Beliefs for** Barbara: $b^{lex}_{Barbara}(t_B) = ((B, t_y); (C, t_y); (A, t_y))$

- Your type t_y primarily believes in Barbara's rationality.
- However, t_y's secondary and tertiary belief seem counter-intuitive.
- For Barbara, *B* is better than *C*, hence it can be plausible to deem Barbara choosing *B* infinitely more likely than her picking *C*.

Respecting the Opponent's Preferences

Definition

A cautious type t_i of player i ***respects the opponent's preferences***, whenever for every opponent's type t_j deemed possible by t_i, if t_j prefers some choice c_j to some other choice c_j', then t_i deems (c_j, t_j) infinitely more likely than (c_j', t_j).

Intuition:
A player deems better choices of his opponent infinitely more likely than worse choices.

Remark:
Respect of preferences can only be defined for cautious types.

Example: Where to read my book?

		Barbara		
		A	B	C
You	A	$0, 3$	$1, 2$	$1, 1$
	B	$1, 3$	$0, 2$	$1, 1$
	C	$1, 3$	$1, 2$	$0, 1$

- **Type Spaces:** $T_{you} = \{t_y, t_y'\}$ and $T_{Barbara} = \{t_B\}$
- **Beliefs for You:** $b_{you}^{lex}(t_y) = ((A, t_B); (C, t_B); (B, t_B))$ and $b_{you}^{lex}(t_y') = ((A, t_B); (B, t_B); (C, t_B))$
- **Beliefs for Barbara:** $b_{Barbara}^{lex}(t_B) = ((B, t_y); (C, t_y); (A, t_y))$

- Your type t_y does not respect Barbara's preferences.
- Your type t_y' does respect Barbara's preferences.
- Note that if you respect Barbara's preferences, then your unique optimal choice is *C*.

Respect of Preferences and Primary Belief in Rationality

Observation. If i is cautious and respects j's preferences, then i also primarily believes in j's rationality.

- Let t_i be some type that is cautious and respects j's preferences.
- Now, consider some pair (c_j, t_j) that is deemed possible by t_i such that c_j is not optimal for t_j.
- Then, there exists some choice c_j^* that t_j prefers to c_j, and t_i must deem (c_j^*, t_j) infinitely more likely than (c_j, t_j).
- Thus, t_i's primary belief must assign probability 0 to (c_j, t_j).

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Common Full Belief in (Caution & Respect of Preferences)

Definition

A cautious type t_i of player i expresses ***common full belief in (caution & respect of preferences)***, if

- t_i expresses 1-fold full belief in caution and respect of preferences, i.e. t_i only deems possible cautious opponent j's types and respects j's preferences,
- t_i expresses 2-fold full belief in caution and respect of preferences, i.e. t_i only deems possible opponent j's types that only deem possible cautious i's types and that respect i's preferences,
- etc.

Relation to Common Full Belief in (Caution & Primary Belief in Rationality)

Proposition

If a cautious type t_i expresses common full belief in (caution & respect of preferences), then t_i entertains common primary belief in (caution & rationality).

Example: Where to read my book?

		Barbara		
		A	B	C
You	A	$0, 3$	$1, 2$	$1, 1$
	B	$1, 3$	$0, 2$	$1, 1$
	C	$1, 3$	$1, 2$	$0, 1$

- **Type Spaces:** $T_{you} = \{t_y\}$ and $T_{Barbara} = \{t_B\}$
- **Beliefs for You:** $b^{lex}_{you}(t_y) = ((A, t_B); (B, t_B); (C, t_B))$
- **Beliefs for Barbara:** $b^{lex}_{Barbara}(t_B) = ((C, t_y); (B, t_y); (A, t_y))$
- Your type t_y is cautious, and respects Barbara's preferences.
- Barbara's type t_B is cautious, and respects your preferences.
- Thus, t_y expresses common full belief in caution and respect of preferences.
- As choice *C* is optimal for type t_y, you can rationally and cautiously go to Pub *C* under common full belief in (caution & respect of preferences).
- Note that under common primary belief in (caution & rationality), you can rationally and cautiously choose *B* as well as *C*.

Example: Dividing a Pizza

Story

- *You* have ordered a four-sliced pizza with *Barbara*.
- Both simultaneously write down the desired number of slices or simply ”the rest”.
- It is agreed that if the numbers’ sum exceeds four, both will give the pizza to charity and neither gets any slice.
- If both write ”the rest”, then the pizza is divided equally among the two.

Example: Dividing a Pizza

		Barbara					
		0	1	2	3	4	*rest*
You	0	0, 0	0, 1	0, 2	0, 3	0, 4	0, 4
	1	1, 0	1, 1	1, 2	1, 3	0, 0	1, 3
	2	2, 0	2, 1	2, 2	0, 0	0, 0	2, 2
	3	3, 0	3, 1	0, 0	0, 0	0, 0	3, 1
	4	4, 0	0, 0	0, 0	0, 0	0, 0	4, 0
	rest	4, 0	3, 1	2, 2	1, 3	0, 4	2, 2

Example: Dividing a Pizza

Barbara

You

	0	1	2	3	4	*rest*
0	0, 0	0, 1	0, 2	0, 3	0, 4	0, 4
1	1, 0	1, 1	1, 2	1, 3	0, 0	1, 3
2	2, 0	2, 1	2, 2	0, 0	0, 0	2, 2
3	3, 0	3, 1	0, 0	0, 0	0, 0	3, 1
4	4, 0	0, 0	0, 0	0, 0	0, 0	4, 0
rest	4, 0	3, 1	2, 2	1, 3	0, 4	2, 2

- What choices can you rationally and cautiously make under common full belief in (caution & respect of preferences)?
- Your choices 0, 1, and 2 are weakly dominated by claiming the *rest*.
- Hence, if you are cautious, then the *rest* is better for you than 0, 1, or 2.
- Similarly, if you believe Barbara to be cautious, then you believe the *rest* to be better for her than 0, 1, or 2.
- As you respect Barbara's preferences, you deem her choice *rest* infinitely more likely than 0, 1, and 2.
- It is now shown that 4 is then better for you than 3.

Example: Dividing a Pizza

		Barbara					
		0	1	2	3	4	*rest*
You	0	0, 0	0, 1	0, 2	0, 3	0, 4	0, 4
	1	1, 0	1, 1	1, 2	1, 3	0, 0	1, 3
	2	2, 0	2, 1	2, 2	0, 0	0, 0	2, 2
	3	3, 0	3, 1	0, 0	0, 0	0, 0	3, 1
	4	4, 0	0, 0	0, 0	0, 0	0, 0	4, 0
	rest	4, 0	3, 1	2, 2	1, 3	0, 4	2, 2

- Indeed, suppose that you deem Barbara's choice *rest* infinitely more likely than 0, 1, and 2.
- There are four possible ways to do so:
  1. You deem *rest* infinitely more likely than her other choices. Then, 4 is better for you than 3.
  2. You deem 4 and *rest* infinitely more likely than her other choices. Then, 4 is better for you than 3.
  3. You deem 3 and *rest* infinitely more likely than her other choices. Then, 4 is better for you than 3.
  4. You deem 3, 4 and *rest* infinitely more likely than her other choices. Then, 4 is better for you than 3.
- Thus, if you are cautious, believe in Barbara's caution, and respect Barbara's preferences, then **you prefer** *rest* **to 0, 1, and 2** and **you prefer 4 to 3**.
- Consequently, under common full belief in (caution & respect of preferences) only 4 and *rest* can possibly be optimal for you!

Example: Dividing a Pizza

		Barbara					
		0	1	2	3	4	*rest*
You	0	0, 0	0, 1	0, 2	0, 3	0, 4	0, 4
	1	1, 0	1, 1	1, 2	1, 3	0, 0	1, 3
	2	2, 0	2, 1	2, 2	0, 0	0, 0	2, 2
	3	3, 0	3, 1	0, 0	0, 0	0, 0	3, 1
	4	4, 0	0, 0	0, 0	0, 0	0, 0	4, 0
	rest	4, 0	3, 1	2, 2	1, 3	0, 4	2, 2

- Consider the following **lexicographic epistemic model**:
  - **Type Spaces:**

    $T_{you} = \{t_y^4, t_y^r\}$ and $T_{Barbara} = \{t_B^4, t_B^r\}$
  - **Beliefs for You:**

    $b_{you}^{lex}(t_y^4) = ((rest, t_B^r); (1, t_B^r); (4, t_B^r); (3, t_B^r); (2, t_B^r); (0, t_B^r))$

    $b_{you}^{lex}(t_y^r) = ((4, t_B^4); (3, t_B^4); (rest, t_B^4); (2, t_B^4); (1, t_B^4); (0, t_B^4))$
  - **Beliefs for Barbara:**

    $b_B^{lex}(t_B^4) = ((rest, t_y^r); (1, t_y^r); (4, t_y^r); (3, t_y^r); (2, t_y^r); (0, t_y^r))$

    $b_B^{lex}(t_B^r) = ((4, t_y^4); (3, t_y^4); (rest, t_y^4); (2, t_y^4); (1, t_y^4); (0, t_y^4))$
- Both your types are cautious and express common full belief in (caution & respect of preferences).
- As 4 is optimal for t_y^4 and *rest* is optimal for t_y^r, you can rationally as well as cautiously choose 4 and *rest* under common full belief in (caution & respect of preferences)!

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An Important Question

Is it always possible – for any given game – that a player cautiously reasons in line with common full belief in (caution & respect of preferences)?

Example: Hide and Seek

Story

- *You* would like to go to a pub to read your book.
- *Barbara* is going to a pub as well, but *you* forgot to ask her to which one.
- Your only objective is to avoid *Barbara*, since *you* would like to read your book in silence.
- *Barbara* prefers *Pub A* to *Pub B*, and *Pub B* to *Pub C*, and she would also like to talk to *you* (2 additional utils for her).
- Question: Which pub should *you* go to?

Example: Hide and Seek

		Barbara		
		A_B	B_B	C_B
You	A_y	$0, 5$	$1, 2$	$1, 1$
	B_y	$1, 3$	$0, 4$	$1, 1$
	C_y	$1, 3$	$1, 2$	$0, 3$

Example: Hide and Seek

		Barbara		
		A_B	B_B	C_B
You	A_y	0, 5	1, 2	1, 1
	B_y	1, 3	0, 4	1, 1
	C_y	1, 3	1, 2	0, 3

Is common full belief in (caution & respect of preferences) possible in this game?

- Consider some arbitrary cautious lexicographic belief about Barbara's choice, e.g. $(A_B; B_B; C_B)$.
- Given this belief, your preferences are C_y preferred to B_y preferred to A_y.
- Consider a cautious lexicographic belief for Barbara that respects these preferences, e.g. $(C_y; B_y; A_y)$.
- Given this belief, Barbara's preferences are A_B preferred to C_B preferred to B_B.
- Consider a cautious lexicographic belief for you that respects these preferences, e.g. $(A_B; C_B; B_B)$.
- Given this belief, your preferences are B_y preferred to C_y preferred to A_y.
- Consider a cautious lexicographic belief for Barbara that respects these preferences, e.g. $(B_y; C_y; A_y)$.
- Given this belief, Barbara's preferences are B_B preferred to A_B preferred to C_B.
- Consider a cautious lexicographic belief for you that respects these preferences, e.g. $(B_B; A_B; C_B)$.
- Given this belief, your preferences are C_y preferred to A_y preferred to B_y.
- Consider a cautious lexicographic belief for Barbara that respects these preferences, e.g. $(C_y; A_y; B_y)$.
- Given this belief, Barbara's preferences are A_B preferred to C_B preferred to B_B.
- Consider a cautious lexicographic belief for you that respects these preferences, e.g. $(A_B; C_B; B_B)$.

Example: Hide and Seek

		Barbara		
		A_B	B_B	C_B
	A_y	$0, 5$	$1, 2$	$1, 1$
You	B_y	$1, 3$	$0, 4$	$1, 1$
	C_y	$1, 3$	$1, 2$	$0, 3$

- A **sequence of lexicographic beliefs** has thus been formed:
  $(A_B; B_B; C_B) \rightarrow (C_y; B_y; A_y) \rightarrow (A_B; C_B; B_B) \rightarrow (B_y; C_y; A_y) \rightarrow (B_B; A_B; C_B) \rightarrow (C_y; A_y; B_y) \rightarrow (A_B; C_B; B_B)$
- It has entered into a **cylce**:
  $(A_B; C_B; B_B) \rightarrow (B_y; C_y; A_y) \rightarrow (B_B; A_B; C_B) \rightarrow (C_y; A_y; B_y) \rightarrow (A_B; C_B; B_B)$

- This cycle is now transformed into a lexicographic epistemic model.
- **Type Spaces:** $T_{you} = \{t_y, t_y'\}$ and $T_{Barbara} = \{t_B, t_B'\}$
- **Beliefs for** ***You*****:** $b_y^{lex}(t_y) = ((A_B, t_B); (C_B, t_B); (B_B, t_B))$ and $b_y^{lex}(t_y') = ((B_B, t_B'); (A_B, t_B'); (C_B, t_B'))$
- **Beliefs for** ***Barbara*****:** $b_B^{lex}(t_B) = ((C_y, t_y'); (A_y, t_y'); (B_y, t_y'))$ and $b_B^{lex}(t_B') = ((B_y, t_y); (C_y, t_y); (A_y, t_y))$

- All types in the epistemic model are cautious and respect the opponent's preferences.
- Hence, all express common full belief in (caution & respect of preferences).
- Concluding, caution and common full belief in (caution & respect of preferences) is indeed possible in the *Hide and Seek* game.

Generalizing the Construction for Existence

- Fix some finite game and consider an arbitrary cautious lexicographic belief $b_i^{lex\,1}$ for player i about j's choice.
- Let R_i^1 be the induced preference relation on C_i for player i given this belief.
- Consider some cautious lexicographic belief $b_j^{lex\,2}$ for player j about i's choice that respects the preference relation R_i^1.
- Let R_j^2 be the induced preference relation on C_j for player j given this belief.
- Consider some cautious lexicographic belief $b_i^{lex\,3}$ for player i about j's choice that respects the preference relation R_i^2.
- Let R_i^3 be the induced preference relation on C_i for player i given this belief.
- etc.
- The sequence of lexicographic beliefs thus constructed bears the following property:
  Any element of the sequence satisfies respect of preferences given the preference relation induced by the immediate predecessor lexicographic belief in the sequence.
- Since there are only finitely many choices and the same lexicographic belief can be specified for any recurring preference relation, the sequence of lexicographic beliefs must eventually enter into a cycle of lexicographic beliefs.

From Lexicographic Beliefs to Types

- Suppose some cycle of lexicographic beliefs:
  $b_i^{lex\,1} \to b_j^{lex\,2} \to b_i^{lex\,3} \to \ldots \to b_j^{lex\,K} \to b_i^{lex\,1}$

- This cycle can be transformed into an lexicographic epistemic model:
  - $b_i(t_i^1) = (b_i^{lex\,1}, t_j^K)$
  - $b_j(t_j^2) = (b_j^{lex\,2}, t_i^1)$
  - $b_i(t_i^3) = (b_i^{lex\,3}, t_j^2)$
  - $b_j(t_j^4) = (b_j^{lex\,4}, t_i^3)$
  - etc.

- In such an epistemic model, every type is cautious and respects the opponent's preferences.

- Hence, all types express common full belief in (caution & respect of preferences)!

Existence

Theorem

Let Γ be some finite two player game. Then, there exists a lexicographic epistemic model such that

- *every type in the model is cautious and expresses common full belief in (caution & respect of preferences),*
- *every type in the model deems possible only one opponent's type, and assigns at each lexicographic level probability-1 to one of the opponent's choices.*

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Towards an Algorithm: Elimination of Choices?

- It is very convenient to have an algorithm which computes the choices that can be made rationally under caution and common full belief in (caution & respect of preferences).
- So far algorithms have been presented that iteratively eliminate choices from the game.
- It is now shown that such an algorithm cannot work for common full belief in (caution & respect of preferences).

Example: Spy Game

Story

- *You* would like to go to a pub to read your book.
- *Barbara* is going to a pub as well, but *you* forgot to ask her to which one.
- Your only objective is to avoid *Barbara*, since *you* would like to read your book in silence.
- *Barbara* prefers *Pub A* to *Pub B*, and *Pub B* to *Pub C*.
- Besides, *Barbara* suspects *you* to have an affair and would thus like to spy on you.
- Spying is only possible from *Pub A* to *Pub C*, or vice versa.
- *Barbara* derives additional utility of 3 from spying.
- Question: Which pub should *you* go to?

Example: Spy Game

Barbara

		A_B	B_B	C_B
	A_y	0, 3	1, 2	1, 4
You	B_y	1, 3	0, 2	1, 1
	C_y	1, 6	1, 2	0, 1

Example: Spy Game

		Barbara		
		A_B	B_B	C_B
You	A_y	0, 3	1, 2	1, 4
	B_y	1, 3	0, 2	1, 1
	C_y	1, 6	1, 2	0, 1

- Which pubs can you rationally and cautiously pick under common full belief in (caution & respect of preferences)?
- Barbara prefers A_B to B_B.
- Therefore, you must deem A_B infinitely more likely than B_B.
- Then, you prefer B_y to A_y.
- Hence, you believe that Barbara deems B_y infinitely more likely than A_y.
- Thus, you believe that Barbara prefers B_B to C_B.
- Consequently, you must deem Barbara's choice B_B infinitely more likely than C_B.
- As you deem A_B infinitely more likely than B_B and B_B infinitely more likely than C_B, you can only rationally choose C_y!

Example: Spy Game

		Barbara		
		A_B	B_B	C_B
You	A_y	$0, 3$	$1, 2$	$1, 4$
	B_y	$1, 3$	$0, 2$	$1, 1$
	C_y	$1, 6$	$1, 2$	$0, 1$

- Consider the following **lexicographic epistemic model**:
  - **Type Spaces:**
    $T_{you} = \{t_y\}$ and $T_{Barbara} = \{t_B\}$
  - **Beliefs for You:**
    $b_{you}(t_y) = ((A_B, t_B); (B_B, t_B); (C_B, t_B))$
  - **Beliefs for Barbara:**
    $b_{Barbara}(t_B) = ((C_y, t_y); (B_y, t_y); (A_y, t_y))$
- Both your types are cautious and express common full belief in (caution & respect of preferences).
- As C_y is optimal for t_y, you can indeed rationally and cautiously choose C_y under common full belief in (caution & respect of preferences)!

Example: Spy Game

		Barbara		
		A_B	B_B	C_B
You	A_y	0, 3	1, 2	1, 4
	B_y	1, 3	0, 2	1, 1
	C_y	1, 6	1, 2	0, 1

- **But:** choice C_y cannot be uniquely filtered out by iteratively deleting strictly or weakly dominated choices!
- At a first step, only B_B could be eliminated.
- But then choice B_y could never be eliminated in the resulting reduced game!

Likelihood Orderings

Definition

A ***likelihood ordering*** for player i on j's choice set is a sequence $L_i = (L_i^1; L_i^2; \dots; L_i^K)$, where $\{L_i^1; L_i^2; \dots; L_i^K\}$ forms a partition of C_j.

Interpretation:

- Player i deems all choices in L_i^1 infinitely more likely than all choices in L_i^2; deems all choices in L_i^2 infinitely more likely than all choices in L_i^3; etc.
- Moreover, a likelihood ordering L_i for player i is said to **assume** a set of choices D_j for the opponent j, whenever L_i deems all choices inside D_j infinitely more likely than all choices outside D_j.
- In other words, an assumed set of choices equals the union of some first l levels of a likelihood ordering.

Preference Restrictions

Definition

A ***preference restriction*** for player i is a pair (c_i, A_i), where $c_i \in C_i$ and $A_i \subseteq C_i$.

Interpretation:

- Player i "prefers" at least one choice in A_i to c_i.
  (Note that "prefer" is used intuitively here, it does not correspond to the well-defined notion prefer!)

- Besides, a likelihood ordering L_i for player i is said to **respect a preference restriction** (c_j, A_j) for the opponent j, whenever L_i deems at least one choice in A_j infinitely more likely than c_j.

Example: Spy Game

Story

- *You* would like to go to a pub to read your book.
- *Barbara* is going to a pub as well, but *you* forgot to ask her to which one.
- Your only objective is to avoid *Barbara*, since *you* would like to read your book in silence.
- *Barbara* prefers *Pub A* to *Pub B*, and *Pub B* to *Pub C*.
- Besides, *Barbara* suspects you to have an affair and would thus like to spy on *you*.
- Spying is only possible from *Pub A* to *Pub C*, or vice versa.
- *Barbara* derives additional utility of 3 from spying.
- Question: Which pub should *you* go to?

Example: Spy Game

Barbara

		A_B	B_B	C_B
	A_y	0, 3	1, 2	1, 4
You	B_y	1, 3	0, 2	1, 1
	C_y	1, 6	1, 2	0, 1

Example: Spy Game

		Barbara		
		A_B	B_B	C_B
You	A_y	$0, 3$	$1, 2$	$1, 4$
	B_y	$1, 3$	$0, 2$	$1, 1$
	C_y	$1, 6$	$1, 2$	$0, 1$

- Barbara prefers A_B to B_B.
- It has been shown above that eliminating choice B_B leads to a dead end.
- However, it can be noted that $(B_B, \{A_B\})$ is a preference restriction for Barbara.
- If you respect Barbara's preference restriction $(B_B, \{A_B\})$, then you must deem A_B infinitely more likely than B_B.
- Thus, your likelihood ordering should be one of the followng:
  1. $(\{A_B\}, \{B_B\}, \{C_B\})$
  2. $(\{A_B\}, \{C_B\}, \{B_B\})$
  3. $(\{A_B\}, \{B_B, C_B\})$
  4. $(\{C_B\}, \{A_B\}, \{B_B\})$
  5. $(\{A_B, C_B\}, \{B_B\})$
- If your likelihood ordering is $(\{A_B\}, \{B_B\}, \{C_B\})$ or $(\{A_B\}, \{C_B\}, \{B_B\})$ or $(\{A_B\}, \{B_B, C_B\})$, then you assume Barbara's choice A_B, i.e. you deem A_B infinitely more likely than her other choices.
- In this case, you prefer B_y to A_y, since B_y weakly dominates A_y on $\{A_B\}$.

Example: Spy Game

		Barbara		
		A_B	B_B	C_B
You	A_y	$0, 3$	$1, 2$	$1, 4$
	B_y	$1, 3$	$0, 2$	$1, 1$
	C_y	$1, 6$	$1, 2$	$0, 1$

- Your likelihood ordering should be one of the followng:

  1. $(\{A_B\}, \{B_B\}, \{C_B\})$
  2. $(\{A_B\}, \{C_B\}, \{B_B\})$
  3. $(\{A_B\}, \{B_B, C_B\})$
  4. $(\{C_B\}, \{A_B\}, \{B_B\})$
  5. $(\{A_B, C_B\}, \{B_B\})$

- If your likelihood ordering is $(\{C_B\}, \{A_B\}, \{B_B\})$ or $(\{A_B, C_B\}, \{B_B\})$, then you assume Barbara's choice set $\{A_B, C_B\}$, i.e. you deem A_B and C_B infinitely more likely than her choice B_B.

- In this case, you prefer B_y to A_y, since B_y weakly dominates A_y on $\{A_B, C_B\}$.

- Indeed, every likelihood ordering for you that respects Barbara's preference restriction $(B_B, \{A_B\})$ assumes either $\{A_B\}$ or $\{A_B, C_B\}$, and on both sets your choice A_y is weakly dominated by B_y.

- Hence, Barbara's preference restriction $(B_B, \{A_B\})$ induces the new preference restriction $(A_y, \{B_y\})$ for you.

Example: Spy Game

		Barbara		
		A_B	B_B	C_B
You	A_y	$0, 3$	$1, 2$	$1, 4$
	B_y	$1, 3$	$0, 2$	$1, 1$
	C_y	$1, 6$	$1, 2$	$0, 1$

- So far there are two preference restrictions: $(A_y, \{B_y\})$ and $(B_B, \{A_B\})$.
  - If Barbara respects your preference restriction $(A_y, \{B_y\})$, then she must deem B_y infinitely more likely than A_y.
  - Hence, her likelihood ordering must assume either your choice B_y or the set $\{B_y, C_y\}$.
  - On B_y as well as on $\{B_y, C_y\}$, Barbara's choice C_B is weakly dominated by B_B.
  - Thus, Barbara prefers B_B to C_B, and $(C_B, \{B_B\})$ results as a new preference restriction for Barbara.
- Now the preference restrictions are as follows: $(A_y, \{B_y\})$, $(B_B, \{A_B\})$, and $(C_B, \{B_B\})$.
  - If you respect Barbara's preference restrictions $(B_B, \{A_B\})$ and $(C_B, \{B_B\})$, then your likelihood ordering must be $(A_B; B_B; C_B)$.
  - Hence, you assume the set $\{A_B, B_B\}$.
  - On $\{A_B, B_B\}$, your choice B_y is weakly dominated by C_y.
  - Thus, you prefer C_y to B_y, and $(B_y, \{C_y\})$ results as a new preference restriction for you.
- The resulting preference restrictions are: $(A_y, \{B_y\})$, $(B_y, \{C_y\})$, $(B_B, \{A_B\})$, and $(C_B, \{B_B\})$.
- Then, your only optimal choice is C_y.
- Indeed, C_y also constitutes the only choice you can rationally and cautiously make under common full belief in (caution & respect of preferences).

Implications of Assuming a Set of Choices

Lemma

Suppose that player i is equipped with some lexicographic belief b_i^{lex} about j's choices and that i assumes a set of choices $D_j \subseteq C_j$ for opponent j. If a choice c_i is weakly dominated on D_j by some randomized choice r_i, then i prefers some choice $c_i^* \in$ supp(r_i) to c_i.

Proof

- Suppose that i entertains lexicographic belief $b_i^{lex} = (b_i^1; \ldots; b_i^K)$ on C_j, and assumes $D_j \subseteq C_j$.
- Then, i deems all choices inside D_j infinitely more likely than all choices outside D_j.
- Consequently, there exists some level k^* such that
  1. for every $d_j \in D_j$ there exists $k \leq k^*$ such that $d_j \in supp(b_i^k)$,
  2. for every $c_j \in C_j \setminus D_j$ there exists no $k \leq k^*$ such that $c_j \in supp(b_i^k)$.
- Hence, the first k^* levels of b_i^{lex} form a cautious lexicographic belief $b_i^{lex D_j} = (b_i^1; \ldots; b_i^{k^*})$ on D_j.
- As r_i weakly dominates c_i on D_j, it follows that for all $k \leq k^*$ $u_i^k(c_i, b_i^{lex D_j}) \leq v_i^k(r_i, b_i^{lex D_j})$, and, since $b_i^{lex D_j}$ is cautious, there exists some $l \leq k^*$ such that $u_i^l(c_i, b_i^{lex D_j}) < v_i^l(r_i, b_i^{lex D_j})$.
- Since $u_i^k(c_i, b_i^{lex D_j}) \leq v_i^k(r_i, b_i^{lex D_j})$ for all $k \leq k^*$, it is – by Basic-Lemma II – the case for all $k \leq k^*$ that either $u_i^k(c_i, b_i^{lex D_j}) = u_i^k(a_i, b_i^{lex D_j})$ for all $a_i \in \text{supp}_i(b_i^k)$
  or there exists $\hat{a}_i \in \text{supp}_i(b_i^k)$ such that $u_i^k(c_i, b_i^{lex D_j}) < u_i^k(\hat{a}_i, b_i^{lex D_j})$.
- Moreover, as $u_i^l(c_i, b_i^{lex D_j}) < v_i^l(r_i, b_i^{lex D_j})$ for some $l \leq k^*$, there must be some $l^* \leq k^*$ and – by Basic-Lemma I – some $a_i^* \in \text{supp}_i(b_i^{l^*})$ such that $u_i^{l^*}(c_i, b_i^{lex D_j}) < u_i^{l^*}(a_i^*, b_i^{lex D_j})$, and denote the smallest such level by l^{min}.
- As $u_i^k(c_i, b_i^{lex D_j}) = u_i^k(a_i^*, b_i^{lex D_j})$ for all $k < l^{min}$ and $u_i^{l^{min}}(c_i, b_i^{lex D_j}) < u_i^{l^{min}}(a_i^*, b_i^{lex D_j})$, player i prefers choice a_i^* to c_i, which concludes the proof.

Agenda

- Respecting the Opponent's Preferences
- Common Full Belief in (Caution & Respect of Preferences)
- Existence
- Towards an Algorithm
- **Algorithm**

Example: Runaway Bride

Story

- *You* are attending *Barbara's* wedding.
- However, when *Barbara* was supposed to say ”yes“, she suddenly changed her mind and ran away with light speed.
- *You* would like to find her and know that she is hiding in one of the following houses:

$$a \quad \rightleftharpoons \quad b \quad \rightleftharpoons \quad c \quad \rightleftharpoons \quad d \quad \rightleftharpoons \quad e$$

- *Barbara's* mother and grandmother live at a and e, respectively, and will definitely not open the door.
- *Your* utility is 1 if you find her, and 0 otherwise.
- *Barbara's* utility equals simply the distance away from you.

Example: Runaway Bride

		Barbara				
		a	*b*	*c*	*d*	*e*
You	*a*	0, 0	0, 1	0, 2	0, 3	0, 4
	b	0, 1	1, 0	0, 1	0, 2	0, 3
	c	0, 2	0, 1	1, 0	0, 1	0, 2
	d	0, 3	0, 2	0, 1	1, 0	0, 1
	e	0, 4	0, 3	0, 2	0, 1	0, 0

Example: Runaway Bride

		Barbara				
		a_B	b_B	c_B	d_B	e_B
	a_Y	$0, 0$	$0, 1$	$0, 2$	$0, 3$	$0, 4$
	b_Y	$0, 1$	$1, 0$	$0, 1$	$0, 2$	$0, 3$
You	c_Y	$0, 2$	$0, 1$	$1, 0$	$0, 1$	$0, 2$
	d_Y	$0, 3$	$0, 2$	$0, 1$	$1, 0$	$0, 1$
	e_Y	$0, 4$	$0, 3$	$0, 2$	$0, 1$	$0, 0$

- What locations can you rationally and cautiously choose under common full belief in (caution & respect of preferences)?
- Observe that c_B is weakly dominated by $\frac{1}{2}b_B + \frac{1}{2}d_B$ on C_Y.
- Thus, Barbara prefers some choice from $\{b_B, d_B\}$ to c_B by the Lemma, and the preference restriction $(c_B, \{b_B, d_B\})$ for Barbara results.
- Preference restrictions: $(c_B, \{b_B, d_B\})$
  - If you respect Barbara's preference restriction $(c_B, \{b_B, d_B\})$, then you must deem either b_B or d_B infinitely more likely than c_B.
  - Hence, you will assume some set $D_B \subseteq C_B$ which includes b_B or d_B but not c_B.
  - On every such set D_B, your choice c_Y is weakly dominated by $\frac{1}{2}b_Y + \frac{1}{2}d_Y$.
  - Thus, you prefer some choice from $\{b_Y, d_Y\}$ to c_Y by the Lemma, and the preference restriction $(c_Y, \{b_Y, d_Y\})$ for you results.
  - Also, a_Y and e_Y are weakly dominated by c_Y on C_B yielding additional preference restrictions $(a_Y, \{c_Y\})$ and $(e_Y, \{c_Y\})$.

Example: Runaway Bride

		Barbara				
		a_B	b_B	c_B	d_B	e_B
	a_Y	0, 0	0, 1	0, 2	0, 3	0, 4
	b_Y	0, 1	1, 0	0, 1	0, 2	0, 3
You	c_Y	0, 2	0, 1	1, 0	0, 1	0, 2
	d_Y	0, 3	0, 2	0, 1	1, 0	0, 1
	e_Y	0, 4	0, 3	0, 2	0, 1	0, 0

- Preference restrictions: $(c_Y, \{b_Y, d_Y\})$, $(a_Y, \{c_Y\})$, $(e_Y, \{c_Y\})$, and $(c_B, \{b_B, d_B\})$
  - Note that b_B and d_B are weakly dominated by $\frac{3}{4}a_B + \frac{1}{4}e_B$ and $\frac{1}{4}a_B + \frac{3}{4}e_B$, respectively, on C_Y, yielding preference restrictions $(b_B, \{a_B, e_B\})$ and $(d_B, \{a_B, e_B\})$ for Barbara.
- Preference restrictions: $(c_Y, \{b_Y, d_Y\})$, $(a_Y, \{c_Y\})$, $(e_Y, \{c_Y\})$, as well as $(c_B, \{b_B, d_B\})$, $(b_B, \{a_B, e_B\})$, and $(d_B, \{a_B, e_B\})$.
- Therefore, only b_Y and d_Y can possibly be optimal for you, and only a_B and e_B can possibly be optimal for Barbara.

Example: Runaway Bride

		Barbara				
		a_B	b_B	c_B	d_B	e_B
You	a_Y	$0, 0$	$0, 1$	$0, 2$	$0, 3$	$0, 4$
	b_Y	$0, 1$	$1, 0$	$0, 1$	$0, 2$	$0, 3$
	c_Y	$0, 2$	$0, 1$	$1, 0$	$0, 1$	$0, 2$
	d_Y	$0, 3$	$0, 2$	$0, 1$	$1, 0$	$0, 1$
	e_Y	$0, 4$	$0, 3$	$0, 2$	$0, 1$	$0, 0$

- Preference restrictions: $(c_Y, \{b_Y, d_Y\})$, $(a_Y, \{c_Y\})$, $(e_Y, \{c_Y\})$, as well as $(c_B, \{b_B, d_B\})$, $(b_B, \{a_B, e_B\})$, and $(d_B, \{a_B, e_B\})$.
- Consider the following **lexicographic epistemic model**:
  - **Type Spaces:**
    $T_{you} = \{t_y^b, t_y^d\}$ and $T_{Barbara} = \{t_B^a, t_B^e\}$
  - **Beliefs for *You*:**
    $b_{you}^{lex}(t_y^b) = ((a_B, t_B^a); (b_B, t_B^a); (e_B, t_B^a); (c_B, t_B^a); (d_B, t_B^a))$
    $b_{you}^{lex}(t_y^d) = ((e_B, t_B^e); (d_B, t_B^e); (a_B, t_B^e); (c_B, t_B^e); (b_B, t_B^e))$
  - **Beliefs for *Barbara*:**
    $b_B^{lex}(t_B^a) = ((d_Y, t_y^d); (c_Y, t_y^d); (b_Y, t_y^d); (a_Y, t_y^d); (e_Y, t_y^d))$
    $b_B^{lex}(t_B^e) = ((b_Y, t_y^b); (c_Y, t_y^b); (d_Y, t_y^b); (a_Y, t_y^b); (e_Y, t_y^b))$

Example: Runaway Bride

		Barbara				
		a_B	b_B	c_B	d_B	e_B
You	a_Y	0, 0	0, 1	0, 2	0, 3	0, 4
	b_Y	0, 1	1, 0	0, 1	0, 2	0, 3
	c_Y	0, 2	0, 1	1, 0	0, 1	0, 2
	d_Y	0, 3	0, 2	0, 1	1, 0	0, 1
	e_Y	0, 4	0, 3	0, 2	0, 1	0, 0

- All four types are cautious and express common full belief in (caution & respect of preferences).
- As b_Y is optimal for t_y^b and d_Y is optimal for t_y^d, you can rationally as well as cautiously choose house b and d under common full belief in (caution & respect of preferences)!

An Algorithm

Basic Idea: iteratively add preference restrictions to the game!

Perea-Procedure

- **Round 1.** For every player i, add a preference restriction $(c_i, \text{supp}(r_i))$, if in the full game c_i is weakly dominated by some randomized choice r_i.
- **Round 2.** For every player i, restrict to likelihood orderings L_i that respect all preference restrictions for the opponent in round 1. If every such likelihood ordering L_i assumes a set of opponent choices D_j on which c_i is weakly dominated by some randomized choice r_i, then add a preference restriction $(c_i, \text{supp}(r_i))$ for player i, .
- etc, until no further preference restrictions can be added.

The choices that survive this algorithm are the ones that are not part of any preference restriction generated during the complete algorithm.

Note: The order and speed in which preference restrictions are added is not relevant for the choices it returns.

Algorithmic Characterization

Theorem

For all $k \geq 1$, the choices that can rationally be made by a cautious type that expresses up to k-fold full belief in caution and respect of preferences are exactly those choices that survive the first $k+1$ steps of the Perea-Procedure.

Corollary

The choices that can rationally be made by a cautious type that expresses common full belief in (caution & respect of preferences) are exactly those choices that survive the Perea-Procedure.

Example: Take a Seat

Story

- *Barbara* and *you* are the only ones to take an exam.
- Both must choose a seat.
- If both choose the same seat, then with probability 0.5 you get the seat you want, and with probability 0.5 you get the one horizontally next to it.
- In order to pass the exam *you* must be able copy from *Barbara*, and the same applies to her.
- A person can only copy from the other person if seated horizontally next or diagonally behind the latter.

Example: Take a Seat

Story (continued)

- The probabilities of successful copying for the respective seats are given in percentages:
  $a = 0$, $b = 10$, $c = d = 20$, $e = f = 45$, $g = h = 95$
- The objective is to maximize the expected percentage of successful copying.
- Question: What seats can you rationally and cautiously choose under common full belief in (caution & respect of preferences)?

Example: Take a Seat

		Barbara							
		a_B	b_B	c_B	d_B	e_B	f_B	g_B	h_B
You	a_Y	5, 5	0, 10	0, 0	0, 20	0, 0	0, 0	0, 0	0, 0
	b_Y	10, 0	5, 5	0, 20	0, 0	0, 0	0, 0	0, 0	0, 0
	c_Y	0, 0	20, 0	20, 20	20, 20	0, 0	0, 45	0, 0	0, 0
	d_Y	20, 0	0, 0	20, 20	20, 20	0, 45	0, 0	0, 0	0, 0
	e_Y	0, 0	0, 0	0, 0	45, 0	45, 45	45, 45	0, 0	0, 95
	f_Y	0, 0	0, 0	45, 0	0, 0	45, 45	45, 45	0, 95	0, 0
	g_Y	0, 0	0, 0	0, 0	0, 0	0, 0	95, 0	95, 95	95, 95
	h_Y	0, 0	0, 0	0, 0	0, 0	95, 0	0, 0	95, 95	95, 95

Example: Take a Seat

		Barbara							
		a_B	b_B	c_B	d_B	e_B	f_B	g_B	h_B
You	a_Y	$5, 5$	$0, 10$	$0, 0$	$0, 20$	$0, 0$	$0, 0$	$0, 0$	$0, 0$
	b_Y	$10, 0$	$5, 5$	$0, 20$	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$0, 0$
	c_Y	$0, 0$	$20, 0$	$20, 20$	$20, 20$	$0, 0$	$0, 45$	$0, 0$	$0, 0$
	d_Y	$20, 0$	$0, 0$	$20, 20$	$20, 20$	$0, 45$	$0, 0$	$0, 0$	$0, 0$
	e_Y	$0, 0$	$0, 0$	$0, 0$	$45, 0$	$45, 45$	$45, 45$	$0, 0$	$0, 95$
	f_Y	$0, 0$	$0, 0$	$45, 0$	$0, 0$	$45, 45$	$45, 45$	$0, 95$	$0, 0$
	g_Y	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$95, 0$	$95, 95$	$95, 95$
	h_Y	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$95, 0$	$0, 0$	$95, 95$	$95, 95$

- **Round 1.**
  - a_Y is weakly dominated by b_Y on C_B.
  - b_Y is weakly dominated by $\frac{1}{2}c_Y + \frac{1}{2}d_Y$ on C_B.
- With symmetry the preference restrictions $(a_Y, \{b_Y\})$ and $(b_Y, \{c_Y, d_Y\})$ as well as $(a_B, \{b_B\})$ and $(b_B, \{c_B, d_B\})$ obtain.

Example: Take a Seat

		Barbara							
		a_B	b_B	c_B	d_B	e_B	f_B	g_B	h_B
You	a_Y	5, 5	0, 10	0, 0	0, 20	0, 0	0, 0	0, 0	0, 0
	b_Y	10, 0	5, 5	0, 20	0, 0	0, 0	0, 0	0, 0	0, 0
	c_Y	0, 0	20, 0	20, 20	20, 20	0, 0	0, 45	0, 0	0, 0
	d_Y	20, 0	0, 0	20, 20	20, 20	0, 45	0, 0	0, 0	0, 0
	e_Y	0, 0	0, 0	0, 0	45, 0	45, 45	45, 45	0, 0	0, 95
	f_Y	0, 0	0, 0	45, 0	0, 0	45, 45	45, 45	0, 95	0, 0
	g_Y	0, 0	0, 0	0, 0	0, 0	0, 0	95, 0	95, 95	95, 95
	h_Y	0, 0	0, 0	0, 0	0, 0	95, 0	0, 0	95, 95	95, 95

- **Round 2.** preference restrictions: $(a_Y, \{b_Y\}), (b_Y, \{c_Y, d_Y\}), (a_B, \{b_B\}), (b_B, \{c_B, d_B\})$
  - If you respect preference restriction $(a_B, \{b_B\})$, then you must assume some set $D_B \subseteq C_B$ which contains b_B but not a_B.
  - For every such set D_B it holds that d_Y is weakly dominated by c_Y.
  - Moreover, if you respect preference restrictions $(a_B, \{b_B\})$ and $(b_B, \{c_B, d_B\})$, then you must assume some set $D_B \subseteq C_B$ which contains c_B or d_B but not a_B and not b_B .
  - For every such set D_B it holds that c_Y is weakly dominated by $\frac{1}{2}e_Y + \frac{1}{2}f_Y$.

Example: Take a Seat

		Barbara							
		a_B	b_B	c_B	d_B	e_B	f_B	g_B	h_B
You	a_Y	5, 5	0, 10	0, 0	0, 20	0, 0	0, 0	0, 0	0, 0
	b_Y	10, 0	5, 5	0, 20	0, 0	0, 0	0, 0	0, 0	0, 0
	c_Y	0, 0	20, 0	20, 20	20, 20	0, 0	0, 45	0, 0	0, 0
	d_Y	20, 0	0, 0	20, 20	20, 20	0, 45	0, 0	0, 0	0, 0
	e_Y	0, 0	0, 0	0, 0	45, 0	45, 45	45, 45	0, 0	0, 95
	f_Y	0, 0	0, 0	45, 0	0, 0	45, 45	45, 45	0, 95	0, 0
	g_Y	0, 0	0, 0	0, 0	0, 0	0, 0	95, 0	95, 95	95, 95
	h_Y	0, 0	0, 0	0, 0	0, 0	95, 0	0, 0	95, 95	95, 95

- **Round 3.** preference restrictions: $(a_Y, \{b_Y\})$, $(b_Y, \{c_Y, d_Y\})$, $(d_Y, \{c_Y\})$, $(c_Y, \{e_Y, f_Y\})$, $(a_B, \{b_B\})$, $(b_B, \{c_B, d_B\})$, $(d_B, \{c_B\})$, $(c_B, \{e_B, f_B\})$
  - If you respect preference restriction $(d_B, \{c_B\})$, then you must assume some set $D_B \subseteq C_B$ which contains c_B but not d_B.
  - For every such set D_B it holds that e_Y is weakly dominated by f_Y.
  - Moreover, if you respect preference restrictions $(a_B, \{b_B\})$, $(b_B, \{c_B, d_B\})$, $(d_B, \{c_B\})$, $(c_B, \{e_B, f_B\})$, then you must assume some set $D_B \subseteq C_B$ which contains e_B or f_B but not any choice from $\{a_B, b_B, c_B, d_B\}$.
  - For every such set D_B it holds that f_Y is weakly dominated by $\frac{1}{2} g_Y + \frac{1}{2} h_Y$.

Example: Take a Seat

		Barbara							
		a_B	b_B	c_B	d_B	e_B	f_B	g_B	h_B
You	a_Y	5, 5	0, 10	0, 0	0, 20	0, 0	0, 0	0, 0	0, 0
	b_Y	10, 0	5, 5	0, 20	0, 0	0, 0	0, 0	0, 0	0, 0
	c_Y	0, 0	20, 0	20, 20	20, 20	0, 0	0, 45	0, 0	0, 0
	d_Y	20, 0	0, 0	20, 20	20, 20	0, 45	0, 0	0, 0	0, 0
	e_Y	0, 0	0, 0	0, 0	45, 0	45, 45	45, 45	0, 0	0, 95
	f_Y	0, 0	0, 0	45, 0	0, 0	45, 45	45, 45	0, 95	0, 0
	g_Y	0, 0	0, 0	0, 0	0, 0	0, 0	95, 0	95, 95	95, 95
	h_Y	0, 0	0, 0	0, 0	0, 0	95, 0	0, 0	95, 95	95, 95

- **Round 4.** preference restrictions: $(a_Y, \{b_Y\}), (b_Y, \{c_Y, d_Y\}), (d_Y, \{c_Y\}), (c_Y, \{e_Y, f_Y\}), (e_Y, \{f_Y\}), (f_Y, \{g_Y, h_Y\}), (a_B, \{b_B\}), (b_B, \{c_B, d_B\}), (d_B, \{c_B\}), (c_B, \{e_B, f_B\}), (e_B, \{f_B\}), (f_B, \{g_B, h_B\})$
  - If you respect preference restriction $(e_B, \{f_B\})$, then you must assume some set $D_B \subseteq C_B$ which contains f_B but not e_B.
  - For every such set D_B it holds that h_Y is weakly dominated by g_Y.
  - However, note that with preference restrictions $(a_Y, \{b_Y\}), (b_Y, \{c_Y, d_Y\}), (d_Y, \{c_Y\}), (c_Y, \{e_Y, f_Y\}), (e_Y, \{f_Y\}), (f_Y, \{g_Y, h_Y\}), (h_Y, \{g_Y\})$, only your choice g_Y can be optimal!
- Under common full belief in (caution & respect of preferences), you can thus only rationally and cautiously take seat g.

Example: Take a Seat

		Barbara							
		a_B	b_B	c_B	d_B	e_B	f_B	g_B	h_B
You	a_Y	5, 5	0, 10	0, 0	0, 20	0, 0	0, 0	0, 0	0, 0
	b_Y	10, 0	5, 5	0, 20	0, 0	0, 0	0, 0	0, 0	0, 0
	c_Y	0, 0	20, 0	20, 20	20, 20	0, 0	0, 45	0, 0	0, 0
	d_Y	20, 0	0, 0	20, 20	20, 20	0, 45	0, 0	0, 0	0, 0
	e_Y	0, 0	0, 0	0, 0	45, 0	45, 45	45, 45	0, 0	0, 95
	f_Y	0, 0	0, 0	45, 0	0, 0	45, 45	45, 45	0, 95	0, 0
	g_Y	0, 0	0, 0	0, 0	0, 0	0, 0	95, 0	95, 95	95, 95
	h_Y	0, 0	0, 0	0, 0	0, 0	95, 0	0, 0	95, 95	95, 95

- Consider the following **lexicographic epistemic model**:
  - **Type Spaces:** $T_{you} = \{t_Y\}$ and $T_{Barbara} = \{t_B\}$
  - **Beliefs for *You*:**
    $b_{you}(t_Y) = ((g_B, t_B); (h_B, t_B); (f_B, t_B); (e_B, t_B); (c_B, t_B); (d_B, t_B); (b_B, t_B); (a_B, t_B))$
  - **Beliefs for *Barbara*:**
    $b_B(t_B) = ((g_Y, t_Y); (h_Y, t_Y); (f_Y, t_Y); (e_Y, t_Y); (c_Y, t_Y); (d_Y, t_Y); (b_Y, t_Y); (a_Y, t_Y))$

Related Solution Concept of Proper Equilibrium (Myerson, 1978)

Classical Definition

A pair of mixed choices $(\sigma_1, \sigma_2) \in \Delta(C_1) \times \Delta(C_2)$ constitutes a proper equilibrium, if there exists a converging sequence (σ_1^n, σ_2^n) of full support mixed choices such that for all $c_i, c_i' \in C_i$, if $u_i(c_i, \sigma_j^n) < u_i(c_i', \sigma_j^n)$ for some $n \in \mathbb{N}$, then $\lim_{n \to \infty} \frac{\sigma_i^n(c_i)}{\sigma_i^n(c_i')} = 0$.

Epistemic Definition

A pair of beliefs $(\sigma_1, \sigma_2) \in \Delta(C_1) \times \Delta(C_2)$ constitutes a proper equilibrium, if there exists a pair of cautious lexicographic beliefs (b_1^{lex}, b_2^{lex}) such that $b_1^1 = \sigma_2$ as well as $b_2^1 = \sigma_1$ and for all $c_i, c_i' \in C_i$, if $u_i^{lex}(c_i, b_i^{lex}) < u_i^{lex}(c_i', b_i^{lex})$, then b_j^{lex} deems c_i infinitely less likely than c_i'.

Epistemic Conditions:
common full belief in (caution & respect of preferences)
+
some correct beliefs assumption (e.g. “simple belief hierarchies”)