ECON915 Microeconomic Theory

Part A: Introduction to Decision Theory

Problem Set 1

Question 1

Consider the binary relation $x = 2y + 1$ between real numbers $x, y \in \mathbb{R}$.

(i) Determine whether the relation is reflexive.

(ii) Determine whether the relation is symmetric.

(iii) Determine whether the relation is transitive.

Question 2

Let $\star \subseteq X \times X$ be a complete binary relation on some set X. Show that $\star$ is reflexive.

Question 3

Let $\succsim \subseteq X \times X$ be a weak preference relation on some set X and $u : X \to \mathbb{R}$ a real-valued fuction. The function u is said to represent $\succsim$ (and is called a utility function), whenever the following equivalence

$$u(x) \geq u(y), \text{ if and only if, } x \succsim y$$

holds for all $x, y \in X$. Show that u is a utility representation of $\succsim$, if and only if, the following two conditions hold.

(i) For all $x, y \in X$ such that $x \succ y$, it is the case that $u(x) > u(y)$.

(ii) For all $x, y \in X$ such that $x \sim y$, it is the case that $u(x) = u(y)$.

Question 4

Let $\sim \subseteq X \times X$ be an indifference relation on some set X. For all $x \in X$ form the set $I(x) := \{x' \in X : x \sim x'\}$.

(i) Show that for all $x, y \in X$, either $I(x) = I(y)$ or $I(x) \cap I(y) = \emptyset$.

(ii) Show that for all $x \in X$, it is the case that $I(x) \neq \emptyset$.

Question 5

Show that if $\succsim \subseteq X \times X$ is a weak preference relation on some finite set X, then there exists a utility representation.

Question 6

Give an example of a countable set C equipped with a strict preference relation $\succ$ such that $\succ$ cannot be represented by a utility function $u : C \to \mathbb{Z}$ that only assigns integers to the alternatives in C.