Tutorial on Epistemic Game Theory
Part II: Dynamic Games

Andrés Perea

EPICENTER
Research Center for
Epistemic Game Theory

Maastricht University

EASSS, June 19, 2018

Belief Revision

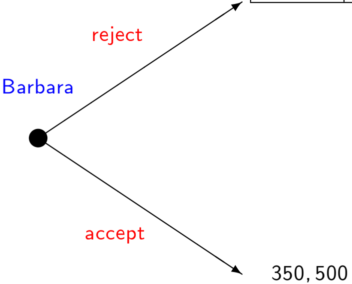
- In a dynamic game, players may choose one after the other.
- Before you make a choice, you may (partially) observe what your opponents have chosen so far.
- It may happen that your initial belief about the opponents' choices will be contradicted later on.
- Then you must revise your belief about the opponents' choices. But how?
- There may be several plausible ways to revise your belief.

Example: Painting Chris' house

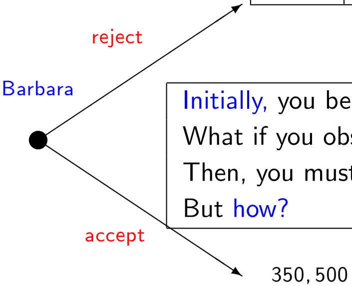
Story

- Chris is planning to paint his house tomorrow, and needs someone to help him.
- You and Barbara are both interested. This evening, both of you must come to Chris' house, and whisper a price in his ear. Price must be either 200, 300, 400 or 500 euros.
- Person with lowest price will get the job. In case of a tie, Chris will toss a coin.
- Before you leave for Chris' house, Barbara gets a phone call from a colleague, who asks her to repair his car tomorrow at a price of 350 euros.
- Barbara must decide whether or not to accept the colleague's offer.

	200	300	400	500
200	$100, 100$	$200, 0$	$200, 0$	$200, 0$
300	$0, 200$	$150, 150$	$300, 0$	$300, 0$
400	$0, 200$	$0, 300$	$200, 200$	$400, 0$
500	$0, 200$	$0, 300$	$0, 400$	$250, 250$

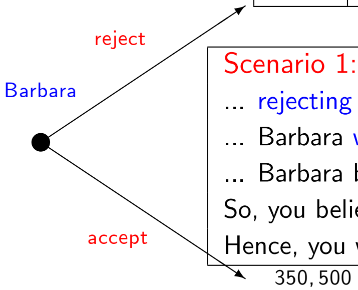


	200	300	400	500
200	100, 100	200, 0	200, 0	200, 0
300	0, 200	150, 150	300, 0	300, 0
400	0, 200	0, 300	200, 200	400, 0
500	0, 200	0, 300	0, 400	250, 250



Initially, you believe that Barbara accepts the offer.
What if you observe that she has rejected the offer?
Then, you must revise your belief.
But how?

	200	300	400	500
200	100, 100	200, 0	200, 0	200, 0
300	0, 200	150, 150	300, 0	300, 0
400	0, 200	0, 300	200, 200	400, 0
500	0, 200	0, 300	0, 400	250, 250



Scenario 1: You believe that ...

... rejecting offer was a mistake by Barbara,

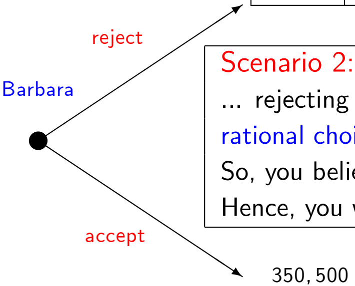
... Barbara will choose rationally from now on

... Barbara believes that you choose rationally.

So, you believe that Barbara chooses 200 or 300.

Hence, you will choose price 200.

	200	300	400	500
200	100, 100	200, 0	200, 0	200, 0
300	0, 200	150, 150	300, 0	300, 0
400	0, 200	0, 300	200, 200	400, 0
500	0, 200	0, 300	0, 400	250, 250



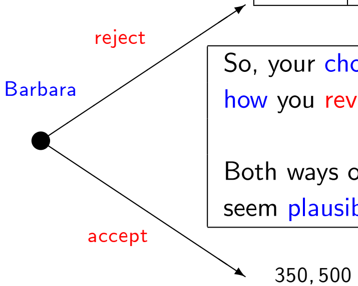
Scenario 2: You believe that ...

... rejecting colleague's offer was a rational choice for Barbara.

So, you believe that Barbara chooses price 400.

Hence, you will choose price 300.

	200	300	400	500
200	100, 100	200, 0	200, 0	200, 0
300	0, 200	150, 150	300, 0	300, 0
400	0, 200	0, 300	200, 200	400, 0
500	0, 200	0, 300	0, 400	250, 250



So, your choice crucially depends on how you revise your belief about Barbara.

Both ways of revising your belief seem plausible.

Dynamic games

- An information set for player i is a situation where player i must make a choice.
- H_i: collection of information sets for player i.
- At an information set h, more than one player can make a choice.

Definition (Strategy)

A strategy for player i is a function s_i that assigns to each of his information sets $h \in H_i$ some available choice $s_i(h)$, unless h cannot be reached due to some choice $s_i(h')$ at an earlier information set $h' \in H_i$.

In the latter case, no choice needs to be specified at h.

- This is different from the classical definition of a strategy!
- Rubinstein (1991) calls this a plan of action.

Epistemic model

- In a dynamic game, you do not only hold a belief once, but you hold a new, conditional belief at each of your information sets.
- You may revise your belief as the game proceeds.
- We would like to model hierarchies of conditional beliefs.
- That is, we want to model
- the conditional belief that player i has, at every information set $h \in H_i$, about his opponents' strategy choices,
- the conditional belief that player i has, at every information set $h \in H_i$, about the conditional belief that opponent j has, at every information set $h' \in H_j$, about the opponents' strategy choices,
- and so on.

- Hence, in a conditional belief hierarchy you hold, at each of your information sets, a conditional belief about
- the opponents' strategy choices, and
- the opponents' conditional belief hierarchies.
- Like before, call a (conditional) belief hierarchy a type.
- Then, a type for you holds, at each of your information sets, a conditional belief about
- the opponents' strategy choices, and
- the opponents' types.
- This leads to an epistemic model.

Definition (Epistemic model)

An epistemic model for a dynamic game specifies for every player i a set T_i of possible types.

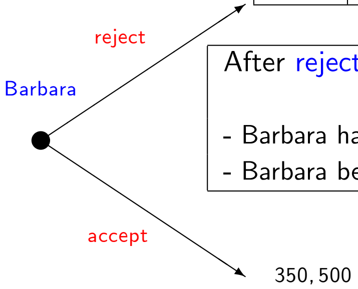
Moreover, every type t_i for player i specifies at every information set $h \in H_i$ a probabilistic belief $b_i(t_i, h)$ over the set $S_{-i}(h) \times T_{-i}$ of opponents' strategy-type combinations.

- Based on Ben-Porath (1997) and Battigalli and Siniscalchi (1999).
- Here, $b_i(t_i, h)$ represents the conditional belief that type t_i holds at information set $h \in H_i$ about the opponents' strategy-type combinations.
- From the epistemic model, we can deduce the complete belief hierarchy for every type.
- A type may revise his belief about the opponents' strategies during the game.
- A type may also revise his beliefs about the opponents' beliefs during the game.

Common belief in future rationality

- We would like to extend the idea of common belief in rationality to dynamic games.
- Problem: At certain information sets, it may not be possible to believe that
- opponent has chosen rationally in the past, or
- opponent has chosen rationally in the past, and that the opponent believes that you choose rationally.
- Hence, common belief in rationality at all information sets is in general not possible.
- We must therefore look for a weaker definition of common belief in rationality.

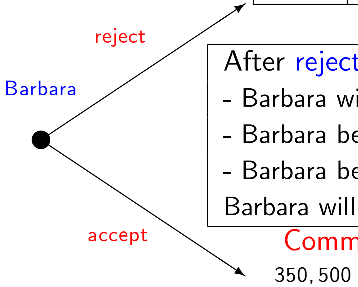
	200	300	400	500
200	100, 100	200, 0	200, 0	200, 0
300	0, 200	150, 150	300, 0	300, 0
400	0, 200	0, 300	200, 200	400, 0
500	0, 200	0, 300	0, 400	250, 250



After reject, you cannot believe that

- Barbara has chosen rationally in the past,
- Barbara believes that you choose rationally.

	200	300	400	500
200	100, 100	200, 0	200, 0	200, 0
300	0, 200	150, 150	300, 0	300, 0
400	0, 200	0, 300	200, 200	400, 0
500	0, 200	0, 300	0, 400	250, 250



After reject, you can believe that
- Barbara will choose rationally in the future,
- Barbara believes that you will choose rationally,
- Barbara believes that you believe that Barbara will choose rationally in the future, etc.

Common belief in future rationality.

- You believe in the opponents' future rationality if you always believe that your opponents will make optimal choices at every present and future information set.

Definition (Belief in the opponents' rationality)

Type t_i believes at h that opponent j chooses rationally at h' if his conditional belief $b_i(t_i, h)$ only assigns positive probability to strategy-type pairs (s_j, t_j) for player j where strategy s_j is optimal for type t_j at information set h'.

Definition (Belief in the opponents' future rationality)

Type t_i believes at h in opponent j's future rationality if t_i believes at h that j chooses rationally at every information set h' for player j that weakly follows h.

Type t_i believes in the opponents' future rationality if t_i believes, at every information set h for player i, in every opponent's future rationality.

- Based on Perea (2014). Similar ideas appear in Baltag, Smets and Zvesper (2009) and Penta (2015).
- Common belief in future rationality means that you always believe that
- your opponents will choose rationally now and in the future,
- your opponents always believe that their opponents will choose rationally now and in the future,
- and so on.

Definition (Common belief in future rationality)

(1) Type t_i expresses 1-fold belief in future rationality if t_i believes in the opponents' future rationality.

(2) Type t_i expresses 2-fold belief in future rationality if t_i assigns, at every information set $h \in H_i$, only positive probability to opponents' types that express 1-fold belief in future rationality.

And so on.

Type t_i expresses common belief in future rationality if t_i expresses k-fold belief in future rationality for every k.

- Based on Perea (2014).
- Similar concepts can be found in Baltag, Smets and Zvesper (2009), Penta (2015), Dekel, Fudenberg and Levine (1999, 2002) and Asheim and Perea (2005).

	200	300	400	500
200	100, 100	200, 0	200, 0	200, 0
300	0, 200	150, 150	300, 0	300, 0
400	0, 200	0, 300	200, 200	400, 0
500	0, 200	0, 300	0, 400	250, 250

reject

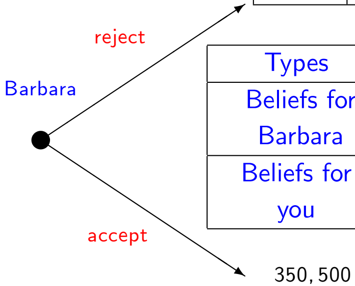
Barbara

Under common belief in future rationality you can only rationally choose 200.

accept

350, 500

200	100, 100	200, 0	200, 0	200, 0
300	0, 200	150, 150	300, 0	300, 0
400	0, 200	0, 300	200, 200	400, 0
500	0, 200	0, 300	0, 400	250, 250



Types	$T_1 = \{t_1\}, \ T_2 = \{t_2\}$
Beliefs for Barbara	$b_1(t_1, \varnothing) = (200, t_2)$ $b_1(t_1, h_1) = (200, t_2)$
Beliefs for you	$b_2(t_2, h_1) = ((reject, 200), t_1)$

Both types express common belief in future rationality.

Recursive Procedure

- We wish to find those strategies that you can rationally choose under common belief in future rationality.
- Can we construct an recursive procedure that helps us find these strategies?
- Yes! It will proceed by iteratedly removing strategies at the various information sets in the game.

- Fix an information set h for player i.
- The full decision problem for player i at h is $\Gamma^0(h) = (S_i(h), S_{-i}(h))$, where $S_i(h)$ is the set of strategies for player i that lead to h, and $S_{-i}(h)$ is the set of opponents' strategy combinations that lead to h.
- A reduced decision problem for player i at h is $\Gamma(h) = (D_i(h), D_{-i}(h))$, where $D_i(h) \subseteq S_i(h)$ and $D_{-i}(h) \subseteq S_{-i}(h)$.

Definition (Backward dominance procedure)

Step 1. At every full decision problem $\Gamma^0(h)$, eliminate for every player i those strategies that are strictly dominated at some full decision problem $\Gamma^0(h')$ that weakly follows h and at which player i is active. This leads to reduced decision problems $\Gamma^1(h)$ at every information set h.

Step 2. At every reduced decision problem $\Gamma^1(h)$, eliminate for every player i those strategies that are strictly dominated at some reduced decision problem $\Gamma^1(h')$ that weakly follows h and at which player i is active. This leads to new reduced decision problems $\Gamma^2(h)$ at every information set.

And so on. Continue until no more strategies can be eliminated in this way.

- Based on Perea (2014).

Definition (Backward dominance procedure)

Step 1. At every full decision problem $\Gamma^0(h)$, eliminate for every player i those strategies that are strictly dominated at some full decision problem $\Gamma^0(h')$ that weakly follows h and at which player i is active. This leads to reduced decision problems $\Gamma^1(h)$ at every information set h.

Step 2. At every reduced decision problem $\Gamma^1(h)$, eliminate for every player i those strategies that are strictly dominated at some reduced decision problem $\Gamma^1(h')$ that weakly follows h and at which player i is active. This leads to new reduced decision problems $\Gamma^2(h)$ at every information set.

And so on. Continue until no more strategies can be eliminated in this way.

- The algorithm always stops within finitely many steps.
- At every information set, it yields a nonempty set of strategies for every player.
- The order in which we eliminate strategies – including the order in which we walk through the information sets – is not important for the final result!

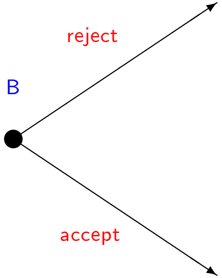
Theorem (Perea (2014))

(1) For every $k \geq 1$, the strategies that can rationally be chosen by a type that expresses up to k-fold belief in future rationality are exactly the strategies that survive the first $k+1$ steps of the backward dominance procedure at $\emptyset$.

(2) The strategies that can rationally be chosen by a type that expresses common belief in future rationality are exactly the strategies that survive the full backward dominance procedure at $\emptyset$.

- Based on Perea (2014).
- A strategy survives the first $k+1$ steps of the backward dominance procedure at $\emptyset$ if it is in the reduced decision problem $\Gamma^{k+1}(\emptyset)$.
- A strategy survives the full backward dominance procedure at $\emptyset$ if it is in the reduced decision problem $\Gamma^{k}(\emptyset)$ for every k.

$\Gamma^0(h_1)$	200	300	400	500
$(r, 200)$	100, 100	200, 0	200, 0	200, 0
$(r, 300)$	0, 200	150, 150	300, 0	300, 0
$(r, 400)$	0, 200	0, 300	200, 200	400, 0
$(r, 500)$	0, 200	0, 300	0, 400	250, 250

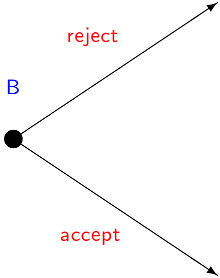


350, 500

$\Gamma^0(\emptyset)$	200	300	400	500
$(r, 200)$	100, 100	200, 0	200, 0	200, 0
$(r, 300)$	0, 200	150, 150	300, 0	300, 0
$(r, 400)$	0, 200	0, 300	200, 200	400, 0
$(r, 500)$	0, 200	0, 300	0, 400	250, 250
accept	350, 500	350, 500	350, 500	350, 500

Step 1

$\Gamma^0(h_1)$	200	300	400	500
$(r, 200)$	100, 100	200, 0	200, 0	200, 0
$(r, 300)$	0, 200	150, 150	300, 0	300, 0
$(r, 400)$	0, 200	0, 300	200, 200	400, 0
$(r, 500)$	0, 200	0, 300	0, 400	250, 250

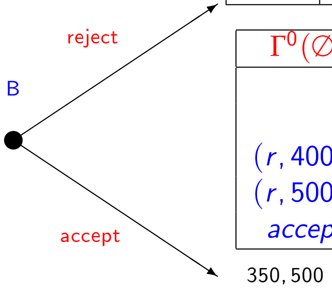


$\Gamma^0(\emptyset)$	200	300	400	500
$(r, 300)$	0, 200	150, 150	300, 0	300, 0
$(r, 400)$	0, 200	0, 300	200, 200	400, 0
$(r, 500)$	0, 200	0, 300	0, 400	250, 250
accept	350, 500	350, 500	350, 500	350, 500

350, 500

Step 1

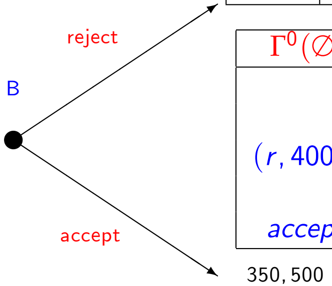
$\Gamma^0(h_1)$	200	300	400	500
$(r, 200)$	100, 100	200, 0	200, 0	200, 0
$(r, 300)$	0, 200	150, 150	300, 0	300, 0
$(r, 400)$	0, 200	0, 300	200, 200	400, 0
$(r, 500)$	0, 200	0, 300	0, 400	250, 250



$\Gamma^0(\emptyset)$	200	300	400	500
$(r, 400)$	0, 200	0, 300	200, 200	400, 0
$(r, 500)$	0, 200	0, 300	0, 400	250, 250
accept	350, 500	350, 500	350, 500	350, 500

Step 1

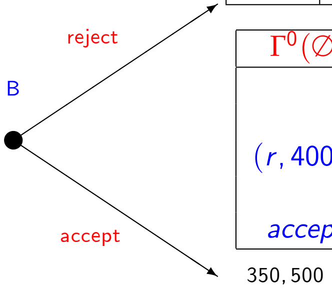
$\Gamma^0(h_1)$	200	300	400	500
$(r, 200)$	100, 100	200, 0	200, 0	200, 0
$(r, 300)$	0, 200	150, 150	300, 0	300, 0
$(r, 400)$	0, 200	0, 300	200, 200	400, 0
$(r, 500)$	0, 200	0, 300	0, 400	250, 250



$\Gamma^0(\emptyset)$	200	300	400	500
$(r, 400)$	0, 200	0, 300	200, 200	400, 0
accept	350, 500	350, 500	350, 500	350, 500

Step 1

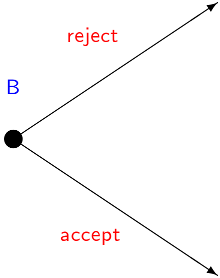
$\Gamma^0(h_1)$	200	300	400	500
$(r, 200)$	100, 100	200, 0	200, 0	200, 0
$(r, 300)$	0, 200	150, 150	300, 0	300, 0
$(r, 400)$	0, 200	0, 300	200, 200	400, 0



$\Gamma^0(\emptyset)$	200	300	400	500
$(r, 400)$	0, 200	0, 300	200, 200	400, 0
accept	350, 500	350, 500	350, 500	350, 500

Step 1

$\Gamma^0(h_1)$	200	300	400	
$(r, 200)$	100, 100	200, 0	200, 0	
$(r, 300)$	0, 200	150, 150	300, 0	
$(r, 400)$	0, 200	0, 300	200, 200	

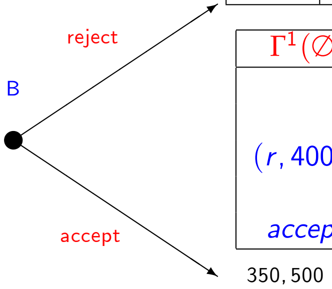


$\Gamma^0(\emptyset)$	200	300	400	500
$(r, 400)$	0, 200	0, 300	200, 200	400, 0
accept	350, 500	350, 500	350, 500	350, 500

350, 500

Step 1

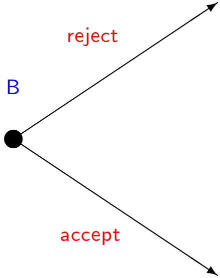
$\Gamma^1(h_1)$	200	300	400	
$(r, 200)$	100, 100	200, 0	200, 0	
$(r, 300)$	0, 200	150, 150	300, 0	
$(r, 400)$	0, 200	0, 300	200, 200	



$\Gamma^1(\emptyset)$	200	300	400
$(r, 400)$	0, 200	0, 300	200, 200
accept	350, 500	350, 500	350, 500

End of Step 1

$\Gamma^1(h_1)$	200	300	400	
$(r, 200)$	$100, 100$	$200, 0$	$200, 0$	
$(r, 300)$	$0, 200$	$150, 150$	$300, 0$	
$(r, 400)$	$0, 200$	$0, 300$	$200, 200$	



$\Gamma^1(\emptyset)$	200	300	400
accept	$350, 500$	$350, 500$	$350, 500$

$350, 500$

Step 2

$\Gamma^1(h_1)$	200	300	400	
$(r, 200)$	$100, 100$	$200, 0$	$200, 0$	
$(r, 300)$	$0, 200$	$150, 150$	$300, 0$	

reject

B

accept

$350, 500$

$\Gamma^1(\emptyset)$	200	300	400
accept	$350, 500$	$350, 500$	$350, 500$

Step 2

$\Gamma^1(h_1)$	200	300		
$(r, 200)$	$100, 100$	$200, 0$		
$(r, 300)$	$0, 200$	$150, 150$		

reject

B

accept

$\Gamma^1(\emptyset)$	200	300	400
accept	$350, 500$	$350, 500$	$350, 500$

$350, 500$

Step 2

$\Gamma^2(h_1)$	200	300		
$(r, 200)$	$100, 100$	$200, 0$		
$(r, 300)$	$0, 200$	$150, 150$		

B

reject

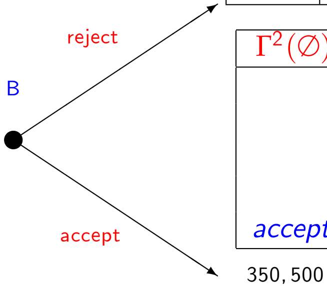
accept

$350, 500$

$\Gamma^2(\varnothing)$	200	300
accept	$350, 500$	$350, 500$

End of Step 2

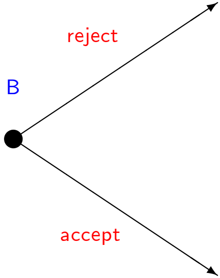
$\Gamma^2(h_1)$	200	300		
$(r, 200)$	$100, 100$	$200, 0$		



$\Gamma^2(\varnothing)$	200	300
accept	$350, 500$	$350, 500$

Step 3

$\Gamma^2(h_1)$	200			
$(r, 200)$	$100, 100$			



$\Gamma^2(\varnothing)$	200	300
accept	$350, 500$	$350, 500$

$350, 500$

Step 3

$\Gamma^3(h_1)$	200			
$(r, 200)$	$100, 100$			

reject

B

accept

$\Gamma^3(\emptyset)$	200
accept	$350, 500$

$350, 500$

End of procedure

Backward induction

- For dynamic games with perfect information, the backward dominance procedure reduces to a very simple procedure called backward induction.

Definition (Game with perfect information)

A dynamic game is with perfect information if at every information set there is only one active player, and this player always knows exactly what choices have been made by his opponents in the past.

Theorem (Common belief in future rationality leads to backward induction)

Consider a dynamic game with perfect information.

Then, the strategies that can rationally be chosen under common belief in future rationality are exactly the backward induction strategies.

Theorem (Common belief in future rationality leads to backward induction)

Consider a dynamic game with perfect information.

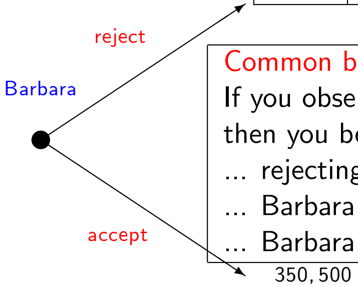
Then, the strategies that can rationally be chosen under common belief in future rationality are exactly the backward induction strategies.

- Hence, common belief in future rationality can be viewed as an epistemic foundation for backward induction.
- Other epistemic foundations for backward induction: Aumann (1995), Samet (1996), Stalnaker (1996, 1998), Balkenborg and Winter (1997), Asheim (2002), Quesada (2002, 2003), Clausing (2003, 2004), Feinberg (2005).
- See Perea (2007) for an overview.

Strong belief in the opponents' rationality

- So far, we have discussed the concept of common belief in future rationality.
- Main idea: Whatever you observe in the game, you always believe that your opponents will choose rationally from now on.
- Common belief in this type of reasoning leads to common belief in future rationality.
- It may not be the only plausible way of reasoning in a dynamic game.

	200	300	400	500
200	100, 100	200, 0	200, 0	200, 0
300	0, 200	150, 150	300, 0	300, 0
400	0, 200	0, 300	200, 200	400, 0
500	0, 200	0, 300	0, 400	250, 250



Common belief in future rationality:

If you observe that Barbara has rejected offer, then you believe that

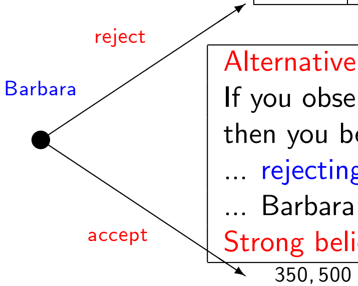
... rejecting offer was a mistake,

... Barbara chooses rationally from now on

... Barbara believes that you choose rationally.

You will choose price 200.

	200	300	400	500
200	100, 100	200, 0	200, 0	200, 0
300	0, 200	150, 150	300, 0	300, 0
400	0, 200	0, 300	200, 200	400, 0
500	0, 200	0, 300	0, 400	250, 250



Alternative way of reasoning:

If you observe that Barbara has rejected offer, then you believe that

... rejecting offer is part of a rational strategy,

... Barbara will choose price 400.

Strong belief in Barbara's rationality.

You will choose price 300.

- Strong belief in the opponents' rationality:
- If at information set $h \in H_i$, it is possible for player i to believe that each of his opponents is implementing a rational strategy,
- then player i must believe at h that each of his opponents is implementing a rational strategy.
- Like belief in the opponents' future rationality, this can be formally defined within an epistemic model.
- To make this possible, the epistemic model must contain sufficiently many types of a certain kind.

	200	300	400	500
200	100, 100	200, 0	200, 0	200, 0
300	0, 200	150, 150	300, 0	300, 0
400	0, 200	0, 300	200, 200	400, 0
500	0, 200	0, 300	0, 400	250, 250

Barbara

reject

accept

350, 500

To formally define **strong belief in Barbara's rationality** the epistemic model must contain **at least** one type for Barbara for which **rejecting** colleague's offer is **optimal.**

- By iterating the condition of strong belief in the opponents' rationality, we arrive at common strong belief in rationality.
- Proposed by Battigalli and Siniscalchi (2002).
- This is a forward induction concept: Whenever possible, you try to explain the past choices made by your opponent.
- In contrast to common belief in future rationality, which is a backward induction concept: You ignore the opponent's past choices, and concentrate solely on the game that lies ahead.
- Battigalli and Siniscalchi (2002) show that common strong belief in rationality characterizes the concept of extensive-form rationalizability (Pearce (1984), Battigalli (1997)).

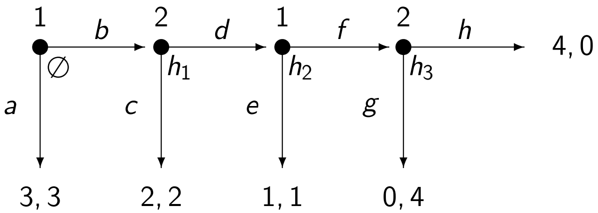
Recursive Procedure

- Shimoji and Watson (1998) proposed the iterated conditional dominance procedure.
- It yields exactly those strategies that can rationally be chosen under common strong belief in rationality.
- Procedure is similar in flavor to the backward dominance procedure.

Comparison with common belief in future rationality

- Common strong belief in rationality and common belief in future rationality represent completely different lines of reasoning.
- The example "Painting Chris' house" has shown that in terms of strategies selected, there is no logical relationship between the two concepts. Both concepts lead to a unique, yet different, strategy choice for you.
- However, both concepts lead to the same outcome in that example, namely that Barbara accepts the colleague's offer at the beginning.
- What about dynamic games with perfect information?

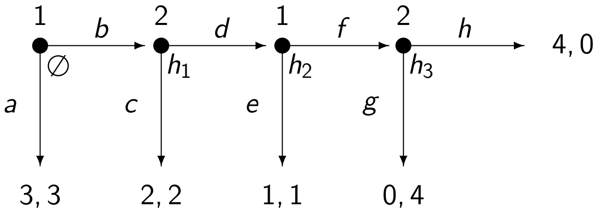
Example: Centipede game.



Common belief in future rationality: Do backward induction.

- At h_3, player 2's backward induction choice is g.
- At h_2, player 1's backward induction choice is e.
- At h_1, player 2's backward induction choice is c.
- At $\varnothing$, player 1's backward induction choice is a.

- Hence, common belief in future rationality uniquely selects strategy c for player 2.
- Induced outcome is a .



- Common strong belief in rationality:
- At h_1, player 2 must believe that player 1 is choosing a rational strategy.
- Hence, at h_1 player 2 must believe that player 1 is implementing the strategy (b, f).
- But then, the unique optimal strategy for player 2 is (d, g).
- Hence, common strong belief in rationality uniquely selects the strategy (d, g) for player 2.
- Induced outcome is a .

Theorem (Outcomes under common strong belief in rationality and common belief in future rationality)

Every outcome that is possible under common strong belief in rationality, is also possible under common belief in future rationality.

- A proof can be found in Perea (2017).
- This result does not hold for strategies.

Theorem (Outcomes under common strong belief in rationality and common belief in future rationality)

Every outcome that is possible under common strong belief in rationality, is also possible under common belief in future rationality.

- Remember that in games with perfect information, common belief in future rationality leads to the backward induction strategies, and hence to the backward induction outcomes.
- In generic games with perfect information, the backward induction outcome is unique.

Corollary (Battigalli's Theorem)

Consider a generic dynamic game with perfect information. Then, the only outcome that is possible under common strong belief in rationality is the backward induction outcome.

- Result does not hold for strategies.

Corollary (Battigalli's Theorem)

Consider a generic dynamic game with perfect information. Then, the only outcome that is possible under common strong belief in rationality is the backward induction outcome.

- This result was first shown by Battigalli (1997).
- Other proofs can be found in Chen and Micali (2013), Heifetz and Perea (2015), Catonini (2017) and Perea (2018).

- Asheim, G.B. (2002), On the epistemic foundation for backward induction, *Mathematical Social Sciences* **44,** 121–144.
- Asheim, G.B. and A. Perea (2005), Sequential and quasi-perfect rationalizability in extensive games, *Games and Economic Behavior* **53**, 15–42.
- Aumann, R. (1995), Backward induction and common knowledge of rationality, *Games and Economic Behavior* **8,** 6–19.
- Balkenborg, D. and E. Winter (1997), A necessary and sufficient epistemic condition for playing backward induction, *Journal of Mathematical Economics* **27,** 325–345.
- Baltag, A., Smets, S. and J.A. Zvesper (2009), Keep 'hoping' for rationality: a solution to the backward induction paradox, *Synthese* **169,** 301–333 (*Knowledge, Rationality and Action* 705–737).

- Battigalli, P. (1997), On rationalizability in extensive games, *Journal of Economic Theory* **74,** 40–61.
- Battigalli, P. and M. Siniscalchi (1999), Hierarchies of conditional beliefs and interactive epistemology in dynamic games, *Journal of Economic Theory* **88**, 188–230.
- Battigalli, P. and M. Siniscalchi (2002), Strong belief and forward induction reasoning, *Journal of Economic Theory* **106,** 356–391.
- Ben-Porath, E. (1997), Rationality, Nash equilibrium and backwards induction in perfect information games, *Review of Economic Studies* **64,** 23–46.
- Catonini, E. (2017), On non-monotonic strategic reasoning, Working paper.
- Chen, J. and S. Micali (2013), The order independence of iterated dominance in extensive games, *Theoretical Economics* **8,** 125–163.

- Clausing, T. (2003), Doxastic conditions for backward induction, *Theory and Decision* **54,** 315–336.
- Clausing, T. (2004), Belief revision in games of perfect information, *Economics and Philosophy* **20,** 89–115.
- Dekel, E., Fudenberg, D. and D.K. Levine (1999), Payoff information and self-confirming equilibrium, *Journal of Economic Theory* **89**, 165–185.
- Dekel, E., Fudenberg, D. and D.K. Levine (2002), Subjective uncertainty over behavior strategies: A correction, *Journal of Economic Theory* **104,** 473–478.
- Feinberg, Y. (2005), Subjective reasoning - dynamic games, *Games and Economic Behavior* **52,** 54–93.
- Heifetz, A. and A. Perea (2015), On the outcome equivalence of backward induction and extensive form rationalizability, *International Journal of Game Theory* **44,** 37–59.

- Pearce, D.G. (1984), Rationalizable strategic behavior and the problem of perfection, *Econometrica* **52,** 1029–1050.
- Perea, A. (2007), Epistemic foundations for backward induction: An overview, *Interactive Logic Proceedings of the 7th Augustus de Morgan Workshop, London. Texts in Logic and Games 1* (Johan van Benthem, Dov Gabbay, Benedikt Löwe (eds.)), Amsterdam University Press, 159–193.
- Perea, A. (2014), Belief in the opponents' future rationality, *Games and Economic Behavior* **83,** 231–254.
- Penta, A. (2015), Robust dynamic implementation, *Journal of Economic Theory* **160,** 280–316.
- Perea, A. (2017), Order independence in dynamic games, *EPICENTER* Working Paper No. 8.
- Perea, A. (2018), Why forward induction leads to the backward induction outcome: A new proof for Battigalli's theorem, *Games and Economic Behavior* **110,** 120–138.

Quesada, A. (2002), Belief system foundations of backward induction, *Theory and Decision* **53,** 393-403.

Quesada, A. (2003), From common knowledge of rationality to backward induction, *International Game Theory Review.*

Rubinstein, A. (1991), Comments on the interpretation of game theory, *Econometrica* **59,** 909–924.

Samet, D. (1996), Hypothetical knowledge and games with perfect information, *Games and Economic Behavior* **17,** 230–251.

Shimoji, M. and J. Watson (1998), Conditional dominance, rationalizability, and game forms, *Journal of Economic Theory* **83**, 161–195.

Stalnaker, R. (1996), Knowledge, belief and counterfactual reasoning in games, *Economics and Philosophy* **12,** 133–163.

Stalnaker, R. (1998), Belief revision in games: forward and backward induction, *Mathematical Social Sciences* **36,** 31–56.