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Monotonicity and equal-opportunity equivalence in bargaining

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Abstract

In this paper, we study two-person bargaining problems represented by a space of alternatives, a status quo point, and the agents' preference relations on the alternatives. The notion of a family of increasing sets is introduced, which reflects a particular way of gradually expanding the set of alternatives. For any given family of increasing sets, we present a solution which is Pareto optimal and monotonic with respect to this family, that is, it makes each agent weakly better off if the set of alternatives is expanded within this family. The solution may be viewed as an expression of equal-opportunity equivalence as defined in Thomson [Soc. Choice Welf. 11 (1994) 137–156]. It is shown to be the unique solution that, in addition to Pareto optimality and the monotonicity property mentioned above, satisfies a uniqueness axiom and unchanged contour independence. A noncooperative bargaining procedure is provided for which the unique backward induction outcome coincides with the solution.
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1. Introduction

This paper deals with two-party disputes, in which both parties hold preferences over a set of alternatives and attempt to reach an agreement on one of them. If the agents fail in reaching an agreement, they fall back to some fixed status quo point. In the remainder, we refer to such situations as bargaining problems. In the axiomatic bargaining literature, there exist two different approaches to this class of problems. Within the first approach, which is sometimes called the welfarist approach, the agents' preferences are represented by utility functions, thus transforming the original bargaining problem into a bargaining problem in utility space, consisting of a set of feasible utility pairs and a status quo utility pair. Axioms and solutions are then formulated entirely within the context of bargaining problems in utility space. Within the second approach, which we refer to as the nonwelfarist approach, the axioms and solutions are defined directly in terms of the original bargaining problem, that is, in terms of the space of alternatives and the agents' preferences.

In this paper, we focus on the combination of two axioms which have played a prominent role in both the welfarist and nonwelfarist approach to bargaining: Pareto optimality and monotonicity. The Pareto optimality axiom is applied in the strong sense, meaning that there should be no other alternative which is weakly preferred by both agents, and strictly preferred by at least one of them. Monotonicity reflects the requirement that none of the agents should be worse off by expanding the opportunities for both agents. The various monotonicity concepts proposed in the literature differ with respect to the possible ways of expansion taken into consideration. The strongest possible version of monotonicity states that no agent should be worse off by increasing the set of possible outcomes in any possible way. Within the domain of bargaining problems in utility space, this property corresponds to strong monotonicity (see, for instance, Kalai, 1977), requiring a solution to be monotonic with respect to every possible expansion of the set of feasible utilities. As it is well-known, strong monotonicity is incompatible with (strong) Pareto optimality. Similar negative results also hold within the nonwelfarist setting. In an abstract sense, each bargaining problem in utility space is mathematically equivalent to a nonwelfarist bargaining problem in which the set of alternatives is the set of feasible utility pairs, and the agents' preferences coincide with the first and second coordinate of the utility pairs, respectively. As such, strong monotonicity and (strong) Pareto optimality are also incompatible on the space of nonwelfarist bargaining problems. Moulin and Thomson (1988) show in Theorem 2 of their paper that within the context of exchange economies, for any small but positive ϵ, there is no allocation rule which is (strongly) Pareto optimal, resource monotonic and gives each agent at least the utility he would get by receiving an ϵ-share of the aggregate endowment. Here, resource monotonicity means that each agent should be weakly better off by increasing the aggregate endowment in an arbitrary fashion.

When insisting on (strong) Pareto optimality and some version of monotonicity, one is thus forced to restrict the possible ways of expansion with respect to which monotonicity is required. For bargaining problems in utility space, several weaker monotonicity concepts have been proposed in this spirit, such as individual monotonicity (Kalai and Smorodinsky, 1975) and global individual monotonicity (Kalai and Rosenthal, 1978). See Thomson and Myerson (1980) for alternative monotonicity properties in utility space.

Well-known monotonicity properties in specific nonwelfarist contexts are, for instance, resource monotonicity for problems of fair division (see, among others, Moulin and Thomson (1988)), technological monotonicity for problems of fair division with production (Chen and Maskin, 1999), and cost-monotonicity for economies with one public good and one private good (Moulin, 1987). In Chun and Thomson (1988), it is analyzed to what extent certain bargaining solutions in utility space, such as the Nash bargaining solution, the Kalai–Smorodinsky solution, the egalitarian solution and the Perles–Maschler solution, satisfy or violate several monotonicity properties when applied to problems of fair division. Each of the above mentioned monotonicity concepts reflects the requirement that no agent should be worse off if the set of feasible alternatives is increased in some particular way(s).

In order to judge whether a given monotonicity property is reasonable, one is in fact led to answer the following question: Which are the possible directions of expanding the agents' opportunities for which it seems reasonable to require that both agents benefit from an expansion in this direction? Intuitively, it seems that both agents should benefit from an expansion only if this expansion is in some sense "fair", that is, if it increases the agents' possibilities in some symmetric way. Whereas such qualitative judgements about the "fairness" of different directions of expansion may be appropriate in specific economic contexts, they seem highly problematic in abstract formulations of a bargaining problem, like the one adopted in this paper, in which a priori, no direction of expansion may be considered more appropriate than another.

Rather than dealing with this delicate issue of the "reasonableness" of a given expansion direction, our focus is shifted towards the following problem: for any particular way of expanding the set of alternatives, is it possible to find a bargaining solution which is both Pareto optimal and monotonic with respect to this way of expansion? Choosing a particular way of expanding the set of alternatives is formalized by the notion of a family of increasing sets $\mathcal{F}$: a correspondence of continuously increasing, nested sets of alternatives. A solution is then called $\mathcal{F}-$monotonic if increasing the set of alternatives within the family $\mathcal{F}$, while leaving the preferences unchanged, is always weakly better for both agents. For any given family of increasing sets $\mathcal{F}$, we present a solution which is both Pareto optimal and $\mathcal{F}-$monotonic. The solution proceeds as follows. For a given set of alternatives A in $\mathcal{F}$, select those Pareto optimal outcomes in A which, for both agents, is equivalent to their best choice in some "reduced" set of alternatives B in $\mathcal{F}$, where B should be the same choice set for both agents. Hence, the solution in A generates the same utilities as a hypothetical situation in which both agents could choose freely from the same reduced set B.

The solution is closely related to the concept of equal-opportunity equivalence defined by Thomson (1994), which combines the ideas of equal opportunities and egalitarian-equivalence in the context of economies with private and public goods. In such environments, an allocation is said to be equal-opportunity equivalent relative to a family of choice sets if there exists some reference set of this family such that each agent is indifferent between the allocation and his best alternative in this reference set. Here, this reference set must be the same for all agents. The solution proposed in this paper thus chooses those Pareto optimal alternatives which are equal-opportunity equivalent with respect to the given family of increasing sets $\mathcal{F}$.

In accordance with the Nash program, we provide an axiomatic characterization of the solution and a mechanism which implements it. The two key axioms in the characterization of the solution are $\mathcal{F}-$monotonicity, which has already been discussed above, and unchanged contour independence. The latter axiom is due to Maniquet (2002) and states that an alternative selected by the solution should remain a solution outcome whenever the agents revise their preferences without changing the upper contour set, lower contour set and indifference set with respect to this alternative. Together with Pareto optimality and a uniqueness property, the two axioms above characterize the solution. The mechanism proposed is a fairly simple sequential move procedure consisting of only two rounds. In contrast to similar mechanisms proposed by Moulin (1984) and Crawford (1979), the role of first mover is given exogenously to one of the players.

The solution we propose has close connections to various existing solutions in the literature. It is shown, for instance, that the solution coincides with the Kalai–Rosenthal solution (Kalai and Rosenthal, 1978) for bargaining problems in utility space after choosing a suitable utility representation of the preferences. It may therefore be interpreted as a nonwelfarist extension of the Kalai–Rosenthal solution, stated in terms of physical outcomes and preferences, instead of utilities. A similar approach can be found, for instance, in Rubinstein et al. (1992), who present nonwelfarist extensions of the Nash bargaining solution and the Kalai–Smorodinsky solution. See also Binmore (1987) and Roemer (1986, 1988), among others, for an approach in which bargaining problems are stated directly in terms of physical outcomes and preferences.

By choosing an appropriate family of increasing sets, the solution generates Pareto efficient egalitarian equivalent allocations (Pazner and Schmeidler, 1978) for pure exchange economies with equal initial endowments. For public good economies with two agents, one public good and one private good, the solution coincides with the egalitarian equivalent cost-sharing method (Moulin, 1987) for a specific choice of the family of increasing sets.

The outline of the paper is as follows. In Section 2, we present the solution and discuss some of its properties. Section 3 provides some applications of the solution to specific economic environments such as exchange economies, public good economies, location problems and resource allocation problems. Sections 4 and 5 deal with the mechanism and the axiomatic characterization, respectively. In Sections 6, we conclude with some brief remarks.

2. Solution and properties

In this section, we introduce a particular class of bargaining solutions and study some of its properties. Before doing so, we formally define the domain of bargaining problems on which the solution operates.

2.1. Bargaining problems

We focus on two-person bargaining problems in which the set of possible outcomes, or alternatives, is given by a nonempty, compact, connected subset A of some Euclidean space $\mathbb{R}^n$. In case the agents do not manage to reach an agreement, they fall back to some status quo point e in A. Both agents hold preference relations $\succeq_1$, $\succeq_2$ on A which are assumed to be representable by continuous utility functions u_1, u_2 on A. By $\succ_i$, we denote the strict preference relation induced by $\succeq_i$, whereas $\sim_i$ is the induced indifference relation. To every bargaining problem described above may thus be assigned a bargaining problem in utility space (S, d) where $S=\{(u_1(a), u_2(a))|a\in A\} \subseteq \mathbb{R}^2$ is the set of feasible utilities and $d=(u_1(e), u_2(e))$ is the utility pair induced by the status quo point. Because A is connected and compact, and u_1, u_2 are continuous, the set S of feasible utilities is connected and compact as well. An alternative $a\in A$ is called Pareto optimal if there is no $a'\in A$ such that $a' \succeq_i a$ for both agents i, and $a' \succ_j a$ for at least one agent j. Similarly, a utility pair $(x, y)\in S$ is said to be Pareto optimal if there is no $(x', y')\in S$ with $x'>x$, $y'\geq y$ or $x'\geq x$, $y'>y$. As a technical assumption, we impose that the set of Pareto optimal utility pairs in S is a connected set in $\mathbb{R}^2$. We refer to this condition as the Pareto connectedness condition, and it is needed to guarantee the existence of the class of solutions to be introduced below. It also plays an important role in the axiomatic characterization and the implementation by a mechanism in Sections 4 and 5. Sufficient conditions for the Pareto connectedness condition to be satisfied are, for instance, that the set S of feasible utilities is convex, as is the usual assumption in bargaining theory, or that the set of Pareto optimal alternatives is connected within A. Note also that Pareto connectedness is purely a condition on the preferences $\succeq_1$, $\succeq_2$, and not on the specific utility representation of the preferences. It may be verified, namely, that Pareto connectedness is equivalent to the following restriction on the preferences: for every two Pareto optimal alternatives a, $b\in A$ and every $c\in A$ with $a\succ_1 c\succ_1 b$ and $a\prec_2 c\prec_2 b$, there are Pareto optimal alternatives d_1, d_2 such that $d_1\sim_1 c$ and $d_2\sim_2 c$. The discussion above is summarized by the following definition.

Definition 1. A two-person bargaining problem is a quadruple $B=(A, e, \succeq_1, \succeq_2)$ where (1) A is the set of alternatives, given by a nonempty, connected compact subset of some Euclidean space $\mathbb{R}^n$, (2) $e\in A$ is the status quo point, (3) $\succeq_1$ and $\succeq_2$ are the agents' preference relations on A, representable by continuous utility functions u_1, u_2 on A and (4) B satisfies the Pareto connectedness condition.

Note that a bargaining problem in our setting is in some sense nonstandard, for there may not exist an alternative in A that Pareto dominates the status quo point e. In fact, the existence of a status quo point is not essential for our analysis, but we decided to include it in our model as to stay close to the classical formulation of bargaining problems.

2.2. Families of increasing sets

As we have mentioned in the Introduction, a key issue in this paper is the concept of monotonicity of bargaining solutions. Intuitively, monotonicity states that, if the set of alternatives is enlarged in some specific way, then both agents should benefit from it. In order to formally define monotonicity in our setup, we should be explicit about the particular way in which the set of alternatives may be increased. To this purpose, we introduce the notion of a family of increasing sets.

Definition 2. A family of increasing sets is a family $\mathcal{F}=\{A(t)|t\in[0,\infty)\}$ of nonempty, connected compact subsets $A(t)$ of some Euclidean space $\mathbb{R}^n$ such that (1) $A(0)=\{e\}$ for some $e\in\mathbb{R}^n$, (2) $A(t)\subseteq A(t')$ whenever $t\leq t'$, and (3) the correspondence $A(\cdot)$ mapping each $t\in[0,\infty)$ to the set $A(t)$ is continuous with respect to the Hausdorff topology.

The family $\mathcal{F}$ is thus a collection of continuously nested sets starting with a single alternative $\{e\}$, and may be viewed as a possible way of enlarging the set of alternatives in a specific economic environment. For instance, $\mathcal{F}$ may be a family of division problems varying in the total amount that may be distributed among the agents. The family $\mathcal{F}$ could also be a collection of location problems for a public good, differing in the size of the area of feasible locations.

The idea of considering families of bargaining problems with gradually increasing outcome spaces may also be found in O'Neill et al. (2004). The difference with our approach is that O'Neill et al. apply the idea to bargaining problems in utility space, considering monotonically increasing sets of feasible utilities rather than increasing sets of alternatives. Their motivation for this approach is that agents usually follow a gradual process in order to reach an agreement. They propose a solution, called the ordinal solution, which assigns an outcome not only to the big problem, but to any nested problem belonging to some fixed sequence of problems.

2.3. A solution

Consider a family $\mathcal{F}=\{A(t)|t\in[0,\infty)\}$ of increasing sets. We propose a correspondence $\varphi^{\mathcal{F}}$ that assigns a solution to every bargaining problem for which the set of alternatives belongs to $\mathcal{F}$. For the definition of $\varphi^{\mathcal{F}}$, and for other purposes in this paper as well, it turns out to be convenient to introduce a universal space of alternatives, and to define the agents' preferences on this universal space. Formally, let

$$A^{\mathcal{F}}=\bigcup_{t\in[0,\infty)}A(t)$$

be the universal space of alternatives induced by the family $\mathcal{F}$ of increasing sets. Hence, $A^{\mathcal{F}}$ is a potentially unbounded set containing all alternatives that are present in the family $\mathcal{F}$. By $\mathcal{B}^{\mathcal{F}}$, we denote the collection of quadruples $B=(A, e, \succeq_1, \succeq_2)$ such that (1) A belongs to $\mathcal{F}$, (2) $\{e\}=A(0)$, (3) $\succeq_1$ and $\succeq_2$ are preference relations on the universal space of alternatives $A^{\mathcal{F}}$, representable by continuous utility functions, and (4) B satisfies the Pareto connectedness condition. Intuitively, $\mathcal{B}^{\mathcal{F}}$ is the set of bargaining problems corresponding to $\mathcal{F}$. The only difference with Definition 1 is that preferences in $\mathcal{B}^{\mathcal{F}}$ are not only defined on the set A of alternatives to which the particular bargaining problem under consideration is restricted but also on the larger, universal space of alternatives $A^{\mathcal{F}}$. A bargaining problem in $\mathcal{B}^{\mathcal{F}}$ is thus given by three parameters: the set $A\in\mathcal{F}$, and the agents' preference relations on $A^{\mathcal{F}}$.

Now, let $\mathcal{B}^*\subseteq\mathcal{B}^{\mathcal{F}}$ be some nonempty subdomain of bargaining problems. A solution on $\mathcal{B}^*$ is a correspondence φ that assigns to every bargaining problem $B=(A, e, \succeq_1, \succeq_2)$ in $\mathcal{B}^*$ a nonempty set of alternatives in A. Because every set $A'\in\mathcal{F}$ is nonempty and compact, and the preferences $\succeq_1, \succeq_2$ on $A^{\mathcal{F}}$ are representable by continuous utility functions, each set $A' \in \mathcal{F}$ contains a maximal element for every agent. For every $A' \in \mathcal{F}$, let $b_i(A')$ be a maximal element for agent i in the set A' with respect to the preference relation $\succeq_i$. We now define the solution $\varphi^{\mathcal{F}}$ on $\mathcal{B}^*$ in the following way.

Definition 3. The solution $\varphi^{\mathcal{F}}$ assigns to every bargaining problem $B=(A, e, \succeq_1, \succeq_2)$ in $\mathcal{B}^*$ the set of alternatives

$$\varphi^{\mathcal{F}}(B) = \{a \in A | a \text{ Pareto optimal and } \exists A' \in \mathcal{F}, A' \subseteq A \text{ such that } a \sim_i b_i(A') \text{ for both } i\}.$$

Hence, in the solution $\varphi^{\mathcal{F}}(B)$ both agents are indifferent between the solution outcome and their best alternative in some reduced set $A' \in \mathcal{F}$, where A' is the same for both agents. The solution $\varphi^{\mathcal{F}}$ may be seen as an application of the equal-opportunity equivalence concept formulated in Thomson (1994), which combines the ideas of equal opportunities and egalitarian-equivalence in the context of economies with private and public goods. In such environments, an allocation $a \in A$ is said to be equal-opportunity equivalent relative to a family $\mathcal{F}$ of choice sets if there exists some set $A' \in \mathcal{F}$ such that every agent i is indifferent between the allocation a and his best choice in A'. Important in the equal-opportunity equivalence notion is that all agents compare the allocation a to the same choice set A'.

A question that remains is which families $\mathcal{F}$ may be viewed appropriate. It seems difficult, if not impossible, to provide a general answer to this question, since the appropriateness of a given family heavily depends on the specific economic environment under consideration. In Section 3, for instance, we choose for each example some specific family $\mathcal{F}$ which we believe is natural in that particular context. We should stress, however, that the basic properties of the solution $\varphi^{\mathcal{F}}$, as well as the axiomatic characterization and the mechanism that implements it, do not depend upon the specific choice of $\mathcal{F}$.

2.4. Properties of the solution

We explore now some basic properties that the solution $\varphi^{\mathcal{F}}$ satisfies. First, we briefly state these properties. We say that a solution φ on $\mathcal{B}^*$ is Pareto optimal if for every bargaining problem B in $\mathcal{B}^*$, it holds that every $a \in \varphi(B)$ is Pareto optimal. The solution φ is called individually rational if for every B and every $a \in \varphi(B)$, it holds that $a \succeq_i e$ for every agent i. We say that φ is nonempty if for every B the set $\varphi(B)$ is nonempty. The solution φ is said to be essentially unique if for every bargaining problem B, the following holds:

(1) for every $a, b \in \varphi(B)$, we have that $a \sim_i b$ for both agents i, and
(2) if $a \in \varphi(B)$ and $b \sim_i a$ for both agents i, then $b \in \varphi(B)$.

Condition (1) is called essentially single-valuedness in Moulin and Thomson (1988), whereas condition (2) coincides with the requirement that a solution be a full correspondence, as stated in Roemer (1988).

Lemma 1. *Let $\mathcal{F}$ be a family of increasing sets, and let $\mathcal{B}^* \subseteq \mathcal{B}^{\mathcal{F}}$ be some nonempty subdomain of bargaining problems. Then, the solution $\varphi^{\mathcal{F}}$ is nonempty, essentially unique, Pareto optimal and individually rational.*

Proof. Let $B=(A(t), e, \succeq_1, \succeq_2)$ be a bargaining problem in $\mathcal{B}^{\mathcal{F}}$. Choose an arbitrary utility representation u_1, u_2 of the agents' preferences. Let (S, d) be the induced bargaining problem in utility space. Denote utility pairs in S by (x, y). Let X and Y be the maximum utility for agents 1 and 2 in S, respectively. Let $Y(X)=\max\{y|(X, y)\in S\}$ and $X(Y)=\max\{x|(x, Y)\in S\}$. Then, $P_1=(X(Y), Y)$ is the Pareto optimal point with the highest utility for agent 2, and $P_2=(X, Y(X))$ is the Pareto optimal point with the highest utility for agent 1. See Fig. 1.

Because (S, d) satisfies the Pareto connectedness condition, the set of Pareto optimal points in S is a strictly decreasing, connected curve which starts at P_1 and ends at P_2. Denote the set of Pareto optimal utility pairs by $PO(S)$. Let $d=(u_1(e), u_2(e))$ be the status quo utility pair, and $Q=(X, Y)$, the pair of utopia utilities. Define the set of utility pairs $S^* = \{(x,y)\in\mathbb{R}^2|u_1(e)\leq x\leq X, u_2(e)\leq y\leq Y\}$. Then, S^* is a rectangle with corner points d, C, Q and D. See Fig. 1.

For every $r\in[0, t]$, let $U_i(r):=u_i(b_i(A(r))$ be agent i's maximal utility in the reduced set of alternatives $A(r)$. Because the utility function u_i is continuous and the family of sets $\{A(r)|r\in[0, t]\}$ is compact valued and continuously increasing, it follows that the function U_i is continuous and weakly increasing in r. Because $A(0)=\{e\}$, we have that $U_i(0)=u_i(e)$, and $U_i(t)=X$ or Y, depending on whether $i=1$ or $i=2$. Consider now the function $U:[0,t]\rightarrow\mathbb{R}^2$ given by $U(r):=(U_1(r), U_2(r))$ for all $r\in[0, t]$. Then, the function U is continuous, weakly increasing in both components, $U(0)=d$, $U(t)=Q$ and $U(r)\in S^*$ for all $r\in[0, t]$.

Because $PO(S)$ is a curve which goes from P_1 to P_2, P_1 is on or above the curve $U(r)$ and P_2 is on or below the curve $U(r)$, we have that $U(r)$ intersects $PO(S)$ in some point. Hence, there is some $r^*\in[0, t]$ such that $U(r^*)=(U_1(r^*), U_2(r^*))\in PO(S)$. We thus have a Pareto optimal alternative $a^*\in A$ such that $u_i(a^*)=u_i(b_i(A(r^*))$ for both i, and hence, $a^*\in\varphi^{\mathcal{F}}(B)$. The solution $\varphi^{\mathcal{F}}$ is thus nonempty.

Essential uniqueness follows from the observation that the weakly increasing curve $U(r)$ intersects the strictly decreasing curve $PO(S)$ in exactly one point. Pareto optimality holds by definition. Individual rationality follows from the observation that the curve $U(r)$ starts at the status quo point d and is weakly increasing. Because the utility pair at the solution $\varphi^{\mathcal{F}}$

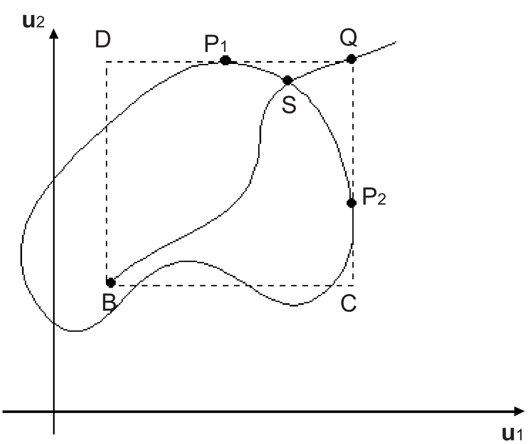

Fig. 1. Existence of solution $\varphi^{\mathcal{F}}$.

is exactly the intersection point between the curve $U(r)$ and the curve $PO(S)$, it follows that the utilities at the solution are as least as high as at the status quo point. □

Note that the proof above could have been performed completely within the space of alternatives, without making use of a particular utility representation. However, the advantage of working within the space of utilities is that geometrical arguments may be used, which facilitates the analysis.

As we have pointed out in the introduction, the aim of this paper is to combine Pareto optimality with some appropriate form of monotonicity. We are now ready to define a notion of monotonicity, based upon the family $\mathcal{F}$ of increasing sets, and prove that it is satisfied by the solution $\varphi^{\mathcal{F}}$.

Definition 4. Let $\mathcal{B}^* \subseteq \mathcal{B}^{\mathcal{F}}$ be a nonempty subdomain of bargaining problems. A solution φ on $\mathcal{B}^*$ is called $\mathcal{F}-$monotonic if for every bargaining problem $B^1 = (A(t), e, \succeq_1, \succeq_2) \in \mathcal{B}^*$, every $r \leq t$ such that $B^2 = (A(r), e, \succeq_1, \succeq_2) \in \mathcal{B}^*$, every $a_1 \in \varphi(B^1)$ and every $a^2 \in \varphi(B^2)$, it holds that $a^1 \succeq_i a^2$ for both agents i.

The $\mathcal{F}-$monotonicity property simply states that enlarging the set of alternatives within the family $\mathcal{F}$ should be beneficial for both agents. We may now prove that the solution $\varphi^{\mathcal{F}}$ satisfies the $\mathcal{F}-$monotonicity property.

Lemma 2. *Let $\mathcal{F}$ be a family of increasing sets and $\mathcal{B}^* \subseteq \mathcal{B}^{\mathcal{F}}$ be some nonempty subdomain of bargaining problems. Then, the solution $\varphi^{\mathcal{F}}$ is $\mathcal{F}-$monotonic.*

Proof. Let $B^1 = (A(t^1), e, \succeq_1, \succeq_2)$, $B^2 = (A(t^2), e, \succeq_1, \succeq_2) \in \mathcal{B}^*$ where $t^1 \leq t^2$. Let $a^{1*} \in \varphi^{\mathcal{F}}(B^1)$ and $a^{2*} \in \varphi^{\mathcal{F}}(B^2)$. We show that $a^{2*} \succeq_i a^{1*}$ for both i. By definition of the solution $\varphi^{\mathcal{F}}$, there are $r^1 \leq t^1$, $r^2 \leq t^2$ such that $a^{1*} \sim_i b_i(A(r^1))$ for both i and $a^{2*} \sim_i b_i(A(r^2))$ for both i. Suppose, now, that $a^{2*} \prec_i a^{1*}$ for some agent i. Because $a^{1*} \sim_i b_i(A(r^1))$ and $a^{2*} \sim_i b_i(A(r^2))$, it follows that $r^2 < r^1$. For the other agent j, we have that $a^{1*} \sim_j b_j(A(r^1))$ and $a^{2*} \sim_j b_j(A(r^2))$. Because $r^2 < r^1$, it follows that $a^{2*} \preceq_j a^{1*}$. However, this contradicts the fact that a^{2*} is Pareto optimal in B^2. Hence, $a^{2*} \succeq_i a^{1*}$ for both i, which implies $\mathcal{F}-$monotonicity. □

We conclude this section by showing that the Pareto connectedness condition is indeed a necessary condition to guarantee the existence of the solution $\varphi^{\mathcal{F}}$ for all possible families of increasing sets.

Example 1. Consider a location problem B in which agents 1 and 2 have to decide upon the location of a public good, somewhere on the interval $A=[-3, 3]$. Let the agents' preferences on the set of alternatives be represented by the utility functions u_1, u_2 where

$$u_1(a) = \max\{1 - |a + 2|, 0\}, \quad u_2(a) = \max\{1 - |a - 2|, 0\}.$$

Hence, agent 1 has a single peak at location -2, and is indifferent between locations in $[-1, 3]$, whereas agent 2 has a single peak at location 2, and is indifferent between locations in $[-3, 1]$. The set of Pareto optimal locations is $\{-2, 2\}$. The set of induced Pareto optimal utility pairs is thus $PO(S)=\{(1, 0), (0, 1)\}$ which is clearly not connected. Hence, this bargaining problem does not satisfy the Pareto connectedness condition.

Consider the family of increasing sets $\mathcal{F} = \{[-t, t] | t \in [0, \infty]\}$. We show that the solution $\varphi^{\mathcal{F}}(B)$ is empty. Suppose that $a^* \in \varphi^{\mathcal{F}}(B)$. Then, because $\varphi^{\mathcal{F}}$ is Pareto optimal, it follows that $a^* \in \{-2, 2\}$. Assume without loss of generality that $a^* = -2$. By definition of $\varphi^{\mathcal{F}}$, there is some $r^* \in [0, 3]$ such that a^* is equivalent, for both agents, to their best choice in $A(r^*) = [-r^*, r^*]$. Because a^* is agent 1's unique maximum, it follows that $a^* \in A(r^*)$, and hence, $r^* \geq 2$. However, agent 2's best choice in $A(r^*)$ is the location 2, which is not equivalent for agent 2 to a^*. Hence, $a^* = -2$ cannot be in $\varphi^{\mathcal{F}}(B)$. Similarly, the location 2 cannot be in $\varphi^{\mathcal{F}}(B)$, which implies that $\varphi^{\mathcal{F}}(B)$ is empty.

Note that it is possible to construct an alternative family of increasing sets $\mathcal{F}'$ for which the solution $\varphi^{\mathcal{F}'}(B)$ is nonempty. Consider, for instance, the (asymmetric) family $\mathcal{F}' = \{[-t/2, 2] | t \in [0, \infty]\}$. It is easy to check that $\varphi^{\mathcal{F}'}(B) = \{2\}$.

3. Applications

Before providing a mechanism which implements the solution $\varphi^{\mathcal{F}}$ and an axiomatic characterization, we wish to illustrate how the solution $\varphi^{\mathcal{F}}$ works in different economic environments. In this section, we apply the solution $\varphi^{\mathcal{F}}$ to pure exchange economies, cost-sharing problems, location problems and resource allocation problems. For each environment, we define a family $\mathcal{F}$ of increasing sets which, in this particular setting, seems to reflect a fair way of enlarging the agents' opportunities. Hence, in each of these examples $\mathcal{F}-$monotonicity may be viewed a desirable property for a solution.

Example 2 (*Pure exchange economies*). Consider a pure exchange economy with two agents and n goods. Suppose that the agents have fixed initial endowments $e^1, e^2 \in \mathbb{R}^n_+$. Let $A = \{(x^1, x^2) \in \mathbb{R}^{n\times 2}_+ | x^1 + x^2 \leq e^1 + e^2\}$ be the set of feasible allocations. By x^i_j, we denote the endowment of good j held by agent i, whereas e^i_j denotes the corresponding initial endowment. For every $t \in [0, 1]$, let

$$\tilde{A}(t) = \{(\tilde{x}^1, \tilde{x}^2) \in \mathbb{R}^{n\times 2}_+ | \tilde{x}^1 + \tilde{x}^2 \leq t(e^1 + e^2)\}$$

be the reduced economy in which only a fraction t of the aggregate endowment is being distributed. By

$$A(t) = \{(x^1, x^2) \in \mathbb{R}^{n\times 2}_+ | \exists (\tilde{x}^1, \tilde{x}^2) \in \tilde{A}(t) \text{s.t.} (x^1, x^2) = (1-t)(e^1, e^2) + (\tilde{x}^1, \tilde{x}^2)\}$$

we denote the reduced exchange economy in which agents 1 and 2 have initial endowments e^1 and e^2, but in which only a fraction t of the aggregate endowment can be exchanged among them. By construction, $A(0) = \{(e^1, e^2)\}$ represents the situation where no trade is possible, and $A(1) = A$. We have thus defined the family of increasing sets $\mathcal{F} = \{(A(t) | t \in [0, 1]\}$[1]. Hence, $\mathcal{F}-$monotonicity in this setting means that increasing the tradable fraction of the aggregate endowment, while using the same fraction for all goods, should be weakly better for both agents.

[1]Formally, a family $\mathcal{F}$ should prescribe a set $A(t)$ for every $t \in [0, \infty)$. Here, we simply set $A(t) = A(1)$ for every $t \geq 1$.

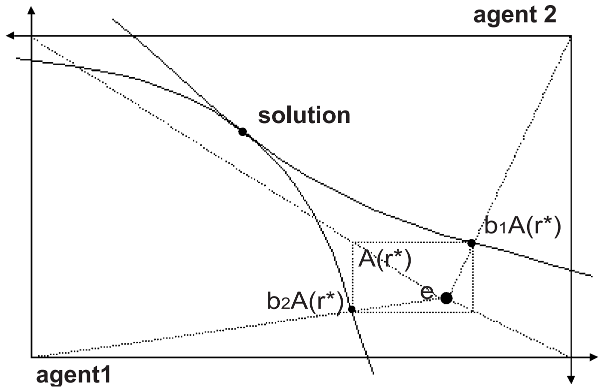

Fig. 2. Solution $\varphi^{\mathcal{F}}$ in exchange economy.

Let $\mathcal{B}^* \subseteq \mathcal{B}^{\mathcal{F}}$ be the domain of all exchange economies $B=(A(t), (e^1, e^2), \succeq_1, \succeq_2)$ where $A(t) \in \mathcal{F}$, the initial endowments (e^1, e^2) are fixed, the preference relations on A are strictly monotonic in all goods, and B satisfies the Pareto connectedness condition. An illustration of the solution $\varphi^{\mathcal{F}}$ for an exchange economy with two goods and strictly convex preferences is given in Fig. 2.

It can be shown, moreover, that this particular solution $\varphi^{\mathcal{F}}$ always generates Pareto efficient egalitarian-equivalent allocations, as defined in Pazner and Schmeidler (1978), if t=1 and both agents have equal initial endowments $e^1=e^2$. Here, egalitarian equivalence states that there should exist a reference bundle (the same for both agents) such that each agent is indifferent between the solution outcome and this reference bundle. In order to show this result, assume the exchange economy is given by some problem $B = \left(A(1), (e^1, e^2), \succeq_1, \succeq_2\right) \in \mathcal{B}^{\mathcal{F}}$, that the aggregate endowment of all goods is normalized to 1, without loss of generality, and that the initial endowment (e^1, e^2) divides the aggregate endowment equally among the agents, that is, $e^1=e^2=(1/2)\mathbf{1}$, where $\mathbf{1}$ denotes the individual bundle containing one unit of all goods. By assumption, both agents have continuous preferences which are monotonic in each good. The solution $\varphi^{\mathcal{F}}(B)$ selects a Pareto optimal allocation (x^1, x^2) and (implicitly) an $r^* \in [0, 1]$ such that each agent i is indifferent between x^i and his best allocation in the reduced economy $A(r^*)$. Now, agent i's best choice from $A(r^*)$ is the bundle $(1-r^*)e^i+r^*\mathbf{1}=(1+r^*)\,(1/2)\,\mathbf{1}$. Hence, both agents are indifferent between the allocation selected by $\varphi^{\mathcal{F}}$ and the egalitarian reference bundle $(1+r^*)(1/2)\,\mathbf{1}$. Because the solution $\varphi^{\mathcal{F}}(B)$ is Pareto optimal, the selected allocation is a Pareto efficient egalitarian-equivalent allocation. The reference bundle, moreover, is a multiple of the equal division bundle (1/2) $\mathbf{1}$, which ensures that the allocation is envy-free (see Pazner and Schmeidler, 1978).[2]

[2]In fact, Pazner and Schmeidler show that choosing the reference bundle equal to a multiple of the equal division bundle is the *only* way to generate envy-free Pareto efficient egalitarian equivalent allocations in all two-agent economies with convex preferences. Pazner and Schmeidler use the term *fair* allocations instead of envy-free allocations.

Example 3 (*Cost sharing in public good economies*). Consider an economy with one public good, one private good and two agents. The public good is produced at a nonnegative level x, using the private good offered by both agents as input. Let y_i be the amount of the private good contributed by agent i. We do not allow for negative contributions. The production technology is given by a function $f(y)$ where $y=y_1+y_2$, and a production capacity $t\in[0, T]$ which means that at most t units can be produced. Here, f is a continuous, nondecreasing function with $f(0)=0$ and $\limsup_{y\to\infty} f(y)/y<\infty$. Suppose that the production capacity is given by t, and that the agents' initial endowments of the private good are given by Y_1 and Y_2. The set of alternatives in this public good economy is thus given by

$$A(t) = \{(x, y_1, y_2)|0\leq y_i\leq Y_i \text{ for both } i \text{ and } 0\leq x\leq \min\{t, f(y_1 + y_2)\}\}.$$

Suppose that $f(Y_i)\geq T$ for both i, that is, the initial endowment of each agent is sufficient to make the maximum production level feasible.

Consider the family of increasing sets $\mathcal{F} = \{A(t)|t\in[0, T]\}$ parametrized by the production capacity t. Let $\mathcal{B}^*$ be the domain of all public good economies $B=(A(t), e, \succeq_1, \succeq_2)$ where (1) $A(t)\in\mathcal{F}$, (2) the status quo point e is (0, 0, 0), (3) the preference relations on $A^{\mathcal{F}} = A(T)$ are representable by continuous utility functions u_i: $A(T)\to\mathbb{R}$ where $u_i(x, y_1, y_2)$ only depends on x and y_i, is nondecreasing in x and decreasing in y_i and (4) B satisfies the Pareto connectedness condition. In this particular setting, $\mathcal{F}-$monotonicity means that increasing the production capacity t makes both agents weakly better off. By construction, $\varphi^{\mathcal{F}}(B)$ chooses those Pareto optimal production-contribution schemes (x, y_1, y_2) in $A(t)$ for which there is some production capacity $r^*\leq t$ such that both agents are indifferent between (x, y_i) and their best choice in $A(r^*)$. Obviously, agent 1's best choice in $A(r^*)$ is $(r^*, 0, f^{-1}(r^*))$, whereas agent 2's best choice in $A(r^*)$ is $(r^*, f^{-1}(r^*), 0)$. Note that $f^{-1}(r^*)$ is feasible for both agents because by assumption $f(Y_i)\geq T\geq r^*$. As such, $\varphi^{\mathcal{F}}(B)$ selects those Pareto optimal (x, y_1, y_2) such that there exists some production level r^* for which $u_i(x, y_i)=u_i(r^*, 0)$ for both i. Hence, both agents are indifferent between the proposed production-contribution scheme and consuming the public good at production level r^* for free. However, this implies that $\varphi^{\mathcal{F}}(B)$ coincides with the egalitarian-equivalent cost sharing method proposed by Moulin (1987), and the value r^* corresponding to the solution $\varphi^{\mathcal{F}}(B)$ is exactly the egalitarian-equivalent production level defined in the same paper. For this equivalence to hold, it is crucial that we do not allow for negative contributions. Moulin (1987), on the other hand, allows for negative contributions in his model, but uses the No Private Transfers axiom stating that the solution should exclude negative contributions.

Example 4 (*Location problems*). Suppose two agents should decide where to locate a public facility. Let $e\in\mathbb{R}^2$ be the location where the facility will be built if agents are not able to reach an agreement. Suppose that the facility has to be located within a radius t from e. So, the set $A(t)$ of possible locations is equal to $\{x\in\mathbb{R}^2|d(x, e)\leq t\}$, where d is the Euclidean distance. Consider the family of increasing sets $\mathcal{F} = \{(A(t)|t\in[0, \infty)\}$ parametrized by the radius t within which the facility has to be built. Let $\mathcal{B}^*$ be the domain of all location problems $B=(A(t), e, \succeq_1, \succeq_2)$ where (1) $A(t)\in\mathcal{F}$, (2) the status quo location e is fixed, (3) for both preference relations $\succeq_i$ there is a most preferred

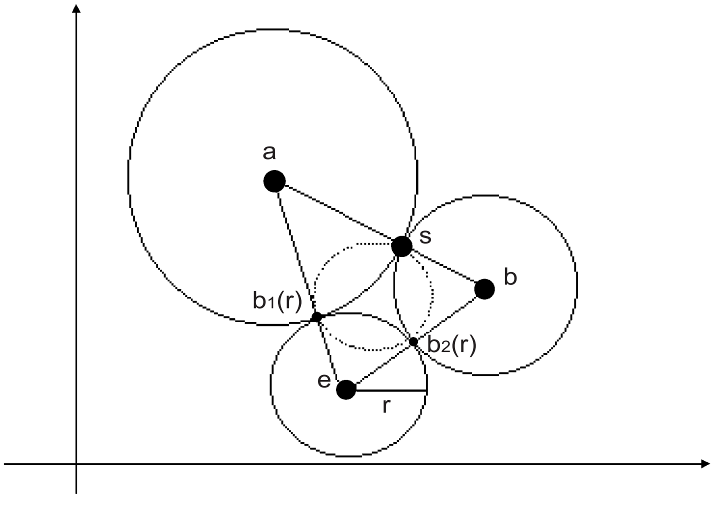

Fig. 3. Solution $\varphi^{\mathcal{F}}$ in location problem.

location $p_i \in \mathbb{R}^2$ such that $x \succeq_i y$ whenever $d(x, p_i) \leq d(y, p_i)$, and (4) B satisfies the Pareto connectedness condition. In order to provide some intuition for the solution $\varphi^{\mathcal{F}}$ in this particular class of problems, let us explicitly determine the selected location for a given location problem $B=(A(t), e, \succeq_1, \succeq_2) \in \mathcal{B}^*$. Let a and b be two feasible locations in $A(t)$ denoting the peaks for agents 1 and 2, respectively, and assume that $\succeq_1$, $\succeq_2$ are strictly decreasing in the distance with respect to the peak. For each $r \leq t$, consider the best locations for both agents in $A(r)$, denoted by $b_1(r)$ and $b_2(r)$. The indifference curve for agent i passing through $b_i(r)$ contains exactly the points that have the same distance with respect to the peak as $b_i(r)$. Because the solution is Pareto optimal, it selects the unique r^* for which the intersection point between these indifference curves hits the line connecting the two peaks (the set of Pareto optimal locations). The agreed upon location is exactly this intersection point. Fig. 3 illustrates the solution, denoted by the point s.

There is an interesting geometrical characterization of the solution $\varphi^{\mathcal{F}}$ in this particular location problem.[3] Consider the triangle abe in Fig. 3, connecting the two peaks and the status quo point. We know, from Fig. 3, that

$$d(a, b_1(r)) = d(a, s),$$

$$d(b, b_2(r)) = d(b, s) \text{ and}$$

$$d(e, b_1(r)) = d(e, b_2(r)).$$

But then, the points s, $b_1(r)$ and $b_2(r)$ must lie on the inscribed circle of the triangle abe, which is the dashed circle in Fig. 3. In particular, the solution $\varphi^{\mathcal{F}}(B)$ is the point on the inscribed circle hitting the line connecting the two peaks.

[3]We thank Hans Peters for pointing out this characterization to us.

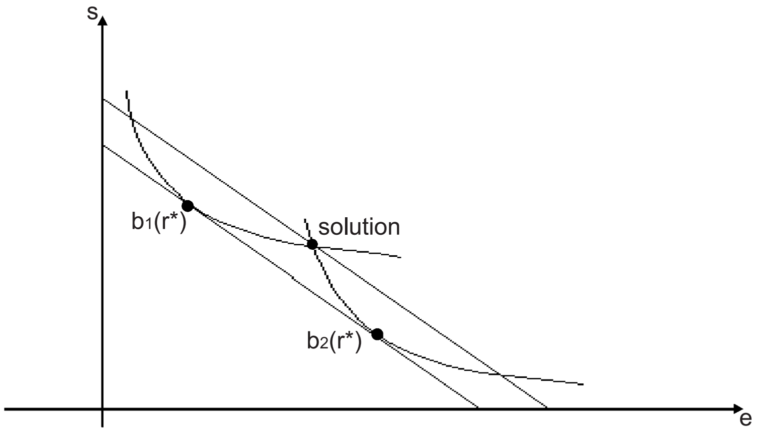

Fig. 4. Solution $\varphi^{\mathcal{F}}$ in resource allocation problem.

Example 5 (*Resource allocation problems*). Consider two scientists involved in a research project, which is financed by the government. Money can be used either for buying new equipment, or for recruiting new staff. Suppose that the total amount of money is t. Let e and s be the amounts of money dedicated to buy equipment and to recruit staff, respectively. So, the set of feasible alternatives is $A(t)=\left\{(e,s)\in\mathbb{R}_+^2|e+s\leq t\right\}$. Suppose that the scientists can spend the money only if they reach an agreement on how to use the research funds. Therefore the status quo point is (0, 0). Consider the family of increasing sets $\mathcal{F}=\{(A(t)|t\in[0,\infty)\}$, parametrized by the total amount of money available. Let $\mathcal{B}^*$ be the domain of all resource allocation problems B=($A(t)$, (0, 0), $\succeq_1$, $\succeq_2$) where (1) $A(t)\in\mathcal{F}$, (2) $\succeq_1$, $\succeq_2$ are weakly increasing in e and s, and (3) B satisfies Pareto connectedness. Here, $\mathcal{F}-$monotonicity means that both agents should be weakly better off if more money is available. In order to provide a graphic intuition of the solution $\varphi^{\mathcal{F}}(B)$ in this context, we consider a resource allocation problem B=($A(t)$, (0, 0), $\succeq_1$, $\succeq_2$), where agents have convex preferences, which are strictly monotonic in both goods. For a given $r\leq t$, consider the optimal distribution of money for the two scientists, $b_1(r)$ and $b_2(r)$, respectively. The solution selects that r^* for which the intersection point between the indifference curves passing through $b_1(r^*)$ and $b_2(r^*)$ hits the line distributing the total amount t of money. See Fig. 4 for an illustration.

4. Mechanism

In this section, we provide a sequential move mechanism with perfect information for which the unique backward induction outcome coincides with the solution $\varphi^{\mathcal{F}}$. It may be verified that the solution $\varphi^{\mathcal{F}}$ is not Maskin monotonic, and hence cannot be implemented in Nash equilibrium by a one-shot mechanism. A mechanism with several rounds is therefore necessary. In order to prove our result, we need to impose some regularity conditions on the bargaining problem at hand.

Definition 5. Let $\mathcal{F} = \{(A(t)|t \in [0, \infty)\}$ be a family of increasing sets. A bargaining problem $B = (A(t), e, \succeq_1, \succeq_2) \in \mathcal{B}^{\mathcal{F}}$ is called regular with respect to $\mathcal{F}$ if

(1) for every $r^1, r^2 \in [0, t]$ with $r^1 < r^2$, it holds that $b_i(A(r^1)) \prec_i b_i(A(r^2))$ for both i and
(2) there is no alternative $a \in A(t)$ with $a \sim_i b_i(A(t))$ for both i.

Condition (1) states that increasing the set of alternatives within $A(t)$ along the family $\mathcal{F}$ strictly increases the utility of the best alternative for both agents. Condition (2) states that the best alternatives for both agents in $A(t)$ differ, thus guaranteeing that there is some conflict of interests. Consider a family of increasing sets $\mathcal{F} = \{(A(t)|t \in [0, \infty)\}$, and some set of alternatives $A(t)$ in $\mathcal{F}$. Let the mechanism $m(\mathcal{F}, t)$ be defined as follows.

Round 1. Agent 1 chooses some number $r \in [0, t]$.
Round 2. Agent 2, after observing r, has two options. He can either choose an alternative a^1 from $A(r)$, after which the mechanism stops and the final outcome is a^1. On the other hand, he can add a single alternative $a^2 \in A(t)$ to $A(r)$ and allow agent 1 to choose the final alternative a^3 from $\{a^2\} \cup A(r)$. In the latter case, the final outcome is a^3.

Hence, in Round 1 agent 1, by choosing an r, decides upon the size of the shrunken pie from which agent 2 may choose, whereas in Round 2, agent 2 may decide to enlarge the shrunken pie by adding an alternative, thereby allowing agent 1 to choose from this larger pie at the end. The mechanism may thus be viewed as a combination of a divide-and-choose and an augment-and-choose procedure, and is similar in spirit to mechanisms proposed by Crawford (1979) and Moulin (1984). However, in contrast to the latter two mechanisms, the first mover in our mechanism is determined exogenously, and not by means of an auction. In Rubinstein's mechanism (Rubinstein, 1982), on the other hand, the role of first mover is assigned exogenously, but a major difference with the mechanism presented here is that in the Rubinstein procedure the amount with which the pie shrinks is exogenously determined by the discount factor, whereas in our mechanism, this amount is chosen by one of the agents.

Theorem 1. *Let $\mathcal{F} = \{(A(t)|t \in [0, \infty)\}$ be a family of increasing sets, and let $B = (A(t), e, \succeq_1, \succeq_2) \in \varphi^{\mathcal{F}}$ be a regular bargaining problem with respect to $\mathcal{F}$. Then, the set of backward induction outcomes of the mechanism $m(\mathcal{F}, t)$ coincides with $\varphi^{\mathcal{F}}(B)$.*

The proof of this theorem is given in the Appendix. Note, finally, that, since we are dealing with a regular bargaining problem, we may transform the curve $U(r)$ in Fig. 1 into a straight line by applying some appropriate monotone transformation of the agents' utility functions. However, by definition, the image of the solution $\varphi^{\mathcal{F}}(B)$ in this particular utility space coincides with the Kalai–Rosenthal solution (Kalai and Rosenthal, 1978). We have thus shown the following result.

Lemma 3. *Let $\mathcal{F} = \{(A(t)|t \in [0, \infty)\}$ be a family of increasing sets, and $B=(A(t), e, \succeq_1, \succeq_2)$ a bargaining problem in $\mathcal{B}^{\mathcal{F}}$ which is regular with respect to $\mathcal{F}$. Then, there exists a utility representation (u_1, u_2) of the preferences such that for the induced bargaining* *problem in utility space (S, d) it holds that* $\varphi^{KR}(S,d) = \{(u_1(a), u_2(a)) | a \in \varphi^{\mathcal{F}}(B)\}$, *where* φ^{KR} *denotes the Kalai–Rosenthal solution.*

5. Axiomatic characterization

In this section, we provide an axiomatic characterization of the solution $\varphi^{\mathcal{F}}$. The solution is characterized by four axioms, Pareto optimality, essential uniqueness, $\mathcal{F}-$monotonicity, and a new property, unchanged contour independence, that may be found in Maniquet (2002). The latter axiom states that a solution outcome should remain a solution outcome if agents revise their preferences while preserving the indifference set, upper contour set and lower contour set with respect to the solution outcome. The axiom is logically weaker than Maskin monotonicity (Maskin, 1977), since the latter property requires a solution to be invariant also against preference transformations that change the upper and lower contour set with respect to the solution outcome. Formally, the axiom is defined as follows. Let $\mathcal{B}^* \subseteq \mathcal{B}^{\mathcal{F}}$ be some nonempty subdomain of bargaining problems.

Definition 6. A solution φ on $\mathcal{B}^*$ is said to satisfy unchanged contour independence if for every bargaining problem $B^1 = (A, e, \succeq_1, \succeq_2) \in \mathcal{B}^*$, every $a^* \in \varphi(B^1)$ and every bargaining problem $B^2 = (A, e, \succeq_1', \succeq_2') \in \mathcal{B}^*$ with

$$[a \succeq_i a^* \text{ if and only if } a \succeq_i' a^*], \quad [a \preceq_i a^* \text{ if and only if } a \preceq_i' a^*]$$

we have that $a^* \in \varphi(B^2)$.

In order for the characterization to hold for a given domain of bargaining problems we have to guarantee that Pareto connectedness is preserved under the admissible preference transformations, and that “enough” preference transformations are being admitted. We call this property $\mathcal{F}$-completeness. In order to introduce the notion of $\mathcal{F}-$completeness of the subdomain $\mathcal{B}^*$, we need the following definitions. We say that $\succeq_i'$ is a lower truncation of the preference relation $\succeq_i$ if there is some alternative $a^* \in A^{\mathcal{F}}$ such that (1) $a \succeq_i' b \succeq_i' a^*$ if and only if $a \succeq_i b \succeq_i a^*$ and (2) $a \preceq_i a^*$ implies $a \sim_i' a^*$. We say that $\succeq_i'$ is an upper truncation of the preference relation $\succeq_i$ if there is some alternative $a^* \in A^{\mathcal{F}}$ such that (1) $a^* \succeq_i' a \succeq_i' b$ if and only if $a^* \succeq_i a \succeq_i b$ and (2) $a \succeq_i a^*$ implies $a \sim_i' a^*$. We call $\succeq_i'$ a truncation of $\succeq_i$ if it is either a lower or an upper truncation of $\succeq_i$.

Definition 7. We say that the domain $\mathcal{B}^*$ is $\mathcal{F}-$complete if the following conditions are satisfied:

(1) if $(A(t), e, \succeq_1, \succeq_2) \in \mathcal{B}^*$, $r \leq t$ and $(A(r), e, \succeq_1, \succeq_2)$ satisfies Pareto connectedness, then $(A(r), e, \succeq_1, \succeq_2) \in \mathcal{B}^*$,
(2) if $(A(t), e, \succeq_1, \succeq_2) \in \mathcal{B}^*$, $\succeq_1', \succeq_2'$ are truncations of $\succeq_1, \succeq_2$ and $(A(t), e, \succeq_1', \succeq_2')$ satisfies Pareto connectedness, then (A(t), e, $\succeq_1', \succeq_2'$)$\in \mathcal{B}^*$.

Now we are ready to state the characterization result.

Theorem 2. *Let $\mathcal{F}$ be a family of increasing sets and $\mathcal{B}^* \subseteq \mathcal{B}^{\mathcal{F}}$ an $\mathcal{F}-$complete domain of bargaining problems. Then, $\varphi^{\mathcal{F}}$ is the only bargaining solution on $\mathcal{B}^*$ that satisfies Pareto optimality, essential uniqueness, $\mathcal{F}-$monotonicity and unchanged contour independence.*

The proof for this result can be found in the Appendix. At this stage, we wish to point out that the $\mathcal{F}-$completeness condition is only needed for the result that the solution $\varphi^{\mathcal{F}}$ is the unique solution on $\mathcal{B}^*$ that satisfies the four axioms above. The other results in this paper, such as the implementation result, the properties listed in Section 2, the fact that the solution satisfies the four axioms above, and the applications to specific economic environments discussed in Section 3, do not depend upon this domain richness condition. Consider, for instance, our Example 2 on exchange economies. When preferences are restricted to be strictly monotonic, the domain will no longer be $\mathcal{F}-$complete, but in spite of this the solution $\varphi^{\mathcal{F}}$ exists, satisfies all the four axioms above, and there is a mechanism that implements it for any possible preference profile in this domain. Note, finally, that in the mechanism, the agents' preference relations are fixed, and hence no preference transformations are needed for the implementation result.

6. Final remarks

In this paper, we have restricted our attention to the case of two agents. A natural question which arises is whether it is possible to extend our analysis to the case of more than two agents. An easy but important first observation is that without putting additional restrictions on the space of bargaining problems, the solution $\varphi^{\mathcal{F}}$, as defined in Section 2, may become empty for more than two agents. Consider, for instance, the family $\mathcal{F} = \{(A(t)|t\in[0, \infty)\}$ of increasing sets of feasible locations as described in Example 4. Suppose that B is a location problem with three agents in which the set of feasible locations is $A(t)$, each agent has quadratic single-peaked preferences, and the peaks of the agents, say respectively a, b and c, lie on the same line. Assume that b is the middle point of the segment $[a, c]$ and that the status quo point e is somewhere below point b, with points a and c having the same distance from e.[4] The set of Pareto optimal locations is the line segment $[a, c]$. As such, the set of Pareto optimal utility triples is a connected set for any utility representation of the agents' preferences, which assures that the problem under consideration satisfies the Pareto connectedness condition applied to three agents. However, it may be easily verified by the reader that there is no Pareto optimal location l and number r such that $l{\sim_i}b_i(A(r))$ for all agents i. Consequently, the solution $\varphi^{\mathcal{F}}$ applied to this location problem is empty. More details on a possible extension of our analysis to the case of more than two agents can be found in an earlier version of this paper, which may be received from the authors upon request.

[4]That is, $d(a, b)=d(b, c)$ and $d(a, e)=d(c, e)$, with $d(e, b)\neq 0$.

Acknowledgements

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Appendix A

Proof of Theorem 1. Let $B=(A(t), e, \succeq_1, \succeq_2)$ be a bargaining problem in $\mathcal{B}^{\mathcal{F}}$ and let $u=(u_1, u_2)$ be an arbitrary utility representation of $(\succeq_1, \succeq_2)$. Let (S, d) be the induced bargaining problem in utility space. We perform the proof within the space (S, d). The following notation is adopted. If we write, for instance, that an agent chooses an alternative $(x, y)\in S$, we mean that he chooses an alternative $a\in A$ with $u_1(a)=x$ and $u_2(a)=y$. By X we denote the highest possible utility for agent 1 in S, whereas Y denotes the maximum utility for agent 2. For every utility x for agent 1, let $Y(x)=\max\{y|(x, y)\in S\}$ be the maximal utility for agent 2 if agent 1's utility is x. In the same way, we define $X(y)$. Let $P_1=(X(Y), Y)$ be the Pareto optimal point with the highest utility for agent 2, and let $P_2=(X, Y(X))$ be the Pareto optimal point with the highest utility for agent 1. Because (S, d) satisfies the Pareto connectedness condition, we know that the Pareto optimal set $PO(S)$ is a strictly decreasing, connected curve from P_1 to P. Or, equivalently, the utility $Y(x)$ is strictly decreasing if $x\geq X(Y)$. For every r, the set $S(r)=\{(u_1(a), u_2(a))|a\in A(r)\}$ is the set of feasible utilities induced by $A(r)$. Let $X(r)$ be the highest possible utility for agent 1 in $S(r)$. Note that $X(r)=u_1(b_1(A(r)))$. Similarly, we define $Y(r)$. Let $r^*=\max\{r\in[0, t]|\,(X(r), Y(r))\in S\}$. Then, the utility pair induced by $\varphi^{\mathcal{F}}(B)$ is $(X(r^*), Y(r^*))$.

We must show that $\varphi^{\mathcal{F}}(B)$ coincides with the backward induction outcomes of the mechanism $m(\mathcal{F}, t)$. Because the proof is performed within utility space, we show that the unique utility pair induced by backward induction is $(X(r^*), Y(r^*))$. To this purpose, we explicitly solve the game $m(\mathcal{F}, t)$ by backward induction.

Round 2. Suppose that agent 1 has chosen some $r\in[0, t]$. We distinguish four cases.

Case 1. Suppose that $X(r)<X(Y)$. In this case, agent 2 can add the new alternative $P_1=(X(Y), Y)$ to $S(r)$. Agent 1 will then certainly choose P_1 because by choosing from $S(r)$, agent 1 can at most get utility $X(r)<X(Y)$. As such, agent 2 can guarantee utility Y, which is the highest utility that agent 2 can possibly achieve. The final outcome in this case is thus some $(x, Y)\in S$ with $x\leq X(Y)$.

Case 2. Suppose that $X(r)\geq X(Y)$ and that $Y(X(r))>Y(r)$. By choosing some alternative $(x^1, y^1)\in S(r)$, agent 2 may achieve a maximum utility of $Y(r)$. If agent 2 adds a new alternative $(x^2, y^2)\in S$ to $S(r)$ then agent 1 will choose (x^2, y^2) if $x^2>X(r)$. Because $Y(X(r))>Y(r)$, there is some $(x^2, y^2)\in S$ with $x^2>X(r)$ and $y^2>Y(r)$ which agent 1 will always choose. By adding this alternative (x^2, y^2), agent 2 can thus guarantee a utility $y^2>Y(r)$. Therefore, the optimal decision for agent 2 in this case is to always add a new alternative. The optimal new alternative (x^2, y^2) which agent 2 can add is one in which $y^2=Y(X(r))$. This follows from the fact that agent 1 can guarantee utility $X(r)$ by choosing his best alternative from $S(r)$, and from the fact that $Y(X(r))$ is strictly decreasing in r if

$X(r) \geq X(Y)$. Because $X(r) \geq X(Y)$ there is exactly one utility pair (x^2, y^2) with $y^2 = Y(X(r))$ and $x^2 \geq X(r)$, namely $(x^2, y^2) = (X(r), Y(X(r)))$. The unique backward induction outcome in this case is thus $(X(r), Y(X(r)))$.

Case 3. Suppose that $X(r) \geq X(Y)$ and that $Y(X(r)) = Y(r)$. By choosing an alternative from $S(r)$ agent 2 can get utility $Y(r)$. If agent 2 would want to add a new alternative (x^2, y^2), agent 2 would choose $(x^2, y^2) = (X(r), Y(X(r))$ which agent 1 would then accept (see Case 2). Because $Y(X(r)) = Y(r)$, agent 2 is indifferent between choosing from $S(r)$ or adding a new alternative. In both cases, agent 2's utility is $Y(r)$. The final outcome in this case is thus some $(x, Y(r)) \in S$ with $x \leq X(Y(r))$.

Case 4. Suppose that $X(r) \geq X(Y)$ and that $Y(X(r)) < Y(r)$. By choosing an alternative from $S(r)$, agent 2 achieves $Y(r)$. If agent 2 adds a new alternative, agent 1 can guarantee $X(r)$ by choosing from $S(r)$. Because $Y(X(r)) < Y(r)$ and $X(r) \geq X(Y)$, there is no $(x, y) \in S$ with $x \geq X(r)$ and $y \geq Y(r)$. Hence, agent 2 strictly prefers to choose from $S(r)$, and the final outcome is thus some $(x, Y(r))$ with $x \leq X(Y(r))$.

Round 1. From the analysis of Round 2, we know the following: (1) by choosing some r with $X(r) < X(Y)$, agent 1 gets at most $X(Y)$, (2) by choosing some r with $X(r) \geq X(Y)$ and $Y(X(r)) > Y(r)$, agent 1 gets exactly $X(r)$, (3) by choosing some r with $X(r) \geq X(Y)$ and $Y(X(r)) \leq Y(r)$, agent 1 gets at most $X(Y(r))$. We show the following claim.

Claim 1. In every backward induction outcome, agent 1 gets at most $X(r^*)$.

Proof of claim 1. Recall that $r^* = \max\{r \in [0, t] | (X(r), Y(r)) \in S\}$. It may be verified easily that $r^* = \max\{r \in [0, t] | Y(X(r)) \geq Y(r)\}$. Hence, the values of r corresponding to (2) satisfy $r \leq r^*$. From (2), we may thus conclude that (2′) by choosing some r with $X(r) \geq X(Y)$ and $Y(X(r)) > Y(r)$, agent 1 gets exactly $X(r) \leq X(r^*)$. Now, let r be such that $X(r) \geq X(Y)$ and $Y(X(r)) \leq Y(r)$. Then, it may be verified that $X(Y(r)) \leq X(r^*)$. From (3) we may thus conclude that (3′) by choosing some r with $X(r) \geq X(Y)$ and $Y(X(r)) \leq Y(r)$, agent 1 gets at most $X(r^*)$. Because $X(r^*) \geq X(Y)$, we may conclude from (1), (2′) and (3′) that agent 1 can get at most $X(r^*)$ in any backward induction outcome. This completes the proof of this claim. □

Claim 2. In every backward induction outcome, agent 1 gets at least $X(r^*)$.

Proof of claim 2. Recall that $PO(S)$ is a strictly decreasing, connected curve from P_1 to P_2 and that $(X(r^*), Y(r^*)) \in PO(S)$. It follows that $X(Y) \leq X(r^*) \leq X$ and $Y(X) \leq Y(r^*) \leq Y$. By assumption, the bargaining problem B is regular with respect to $\mathcal{F}$. We may therefore conclude that the utopia point (X, Y) is not in S and that $X(r)$ and $Y(r)$ are strictly increasing in r. Since $(X, Y) = (X(t), Y(t)) \notin S$ it follows that $r^* < t$ and hence $Y(r^*) < Y(t) = Y$. Hence, there is some $r < r^*$ with $X(r) > X(Y)$ and $Y(X(r)) > Y(r)$. Choose some $\epsilon > 0$. Then, we can find some $r < r^*$ with $X(r) \geq X(r^*) - \epsilon \geq X(Y)$ and $Y(X(r)) > Y(r)$. By choosing this r, we know from (2) that agent 1 gets exactly $X(r) \geq X(r^*) - \epsilon$. Because this holds for every $\epsilon > 0$, agent 1 should get at least $X(r^*)$ in every backward induction outcome. This completes the proof of this claim. □

From Claim 1 and Claim 2, it follows that in every backward induction outcome (if one exists) agent 1 should get exactly $X(r^*)$.

Claim 3. In every backward induction outcome, agent 2 should get exactly $Y(r^*)$.

Proof of claim 3. We distinguish three cases.

Case 1. Suppose that $X(r)<X(Y)$. From case 1 in round 2, we know that agent 2 gets $Y\geq Y(r^*)$.

Case 2. Suppose that $X(r)\geq X(Y)$ and $Y(X(r))>Y(r)$. This implies that $r<r^*$.

From Case 2 in round 2, we know that agent 2 gets $Y(X(r))\geq Y(r^*)$ since $r<r^*$ and $X(r)\geq X(Y)$.

Case 3. Suppose that $X(r)\geq X(Y)$ and $Y(X(r))\leq Y(r)$. This implies that $Y(r)\geq Y(r^*)$. From Cases 3 and 4 in round 2, we know that agent 2 gets $Y(r)\geq Y(r^*)$. □

We may thus conclude that in every backward induction outcome, agent 2 gets at least $Y(r^*)$. Because we already know that agent 1 gets exactly $X(r^*)$, we have on the other hand that agent 2 should get at most $Y(X(r^*))=Y(r^*)$. Hence, agent 2 should get exactly $Y(r^*)$ in every backward induction outcome. This completes the proof of this claim.

From Claims 1, 2 and 3, it follows that there is at most one backward induction outcome in utility space, namely $(X(r^*), Y(r^*))$. It remains to prove that the mechanism has a backward induction strategy profile, which would then yield necessarily the utilities $(X(r^*), Y(r^*))$. Consider the following strategy profile. At round 1, agent 1 chooses r^*. At round 2, agent 2 adds the alternative $(x^2, y^2)=(X(r), Y(X(r))$ to $S(r)$ if $r\leq r^*$, and agent 2 chooses a best alternative from $S(r)$ if $r>r^*$. At round 2, if agent 1 is to choose from $S(r)\cup\{(x^2, y^2)\}$, agent 1 chooses (x^2, y^2) if $x^2\geq X(r)$, and chooses a best alternative from $S(r)$ otherwise. It can easily be checked that this strategy profile satisfies backward induction. Because this strategy profile yields the outcome $(X(r^*), Y(X(r^*)))=(X(r^*), Y(r^*))$, it follows that $(X(r^*), Y(r^*))$ is the unique backward induction outcome of the mechanism $m(\mathcal{F}, t)$ in utility space.

Note that the backward induction strategy profile constructed above in the space of utilities can be reproduced in the space of alternatives. Because every alternative a with $(u_1(a), u_2(a))=(X(r^*), Y(r^*))$ can be obtained as a backward induction outcome in this way, it follows that the set of backward induction outcomes in the space of alternatives is given by $\{a\in A|u_1(a)=X(r^*)$ and $u_2(a)=Y(r^*)\}=\varphi^{\mathcal{F}}(B)$. This completes the proof of this theorem. □

Proof of Theorem 2. First, we show that $\varphi^{\mathcal{F}}$ satisfies the four axioms. From Section 2, we know that $\varphi^{\mathcal{F}}$ is Pareto optimal, essentially unique and $\mathcal{F}$–monotonic. We finally prove unchanged contour independence. Let $B^1=(A, e, \succeq_1, \succeq_2)\in\mathcal{B}^*$, let $a^*\in\varphi^{\mathcal{F}}(B^1)$ and let $B^2=(A, e, \succeq'_1, \succeq'_2)\in\mathcal{B}^*$ with

$$[a\succeq_i a^* \text{ if and only if } a\succeq'_i a^*], \qquad [a\preceq_i a^* \text{ if and only if } a\preceq'_i a^*]$$

for both agents i. We show that $a^*\in\varphi^{\mathcal{F}}(B^2)$. Because $A\in\mathcal{F}$, there is some t with $A=A(t)$. Because $a^*\in\varphi^{\mathcal{F}}(B^1)$, there is some $r\in[0, t]$ with $a^*\sim_i b_i(A(r)|\succeq_i)$ for both i. Here, we write $b_i(A(r)|\succeq_i)$ in order to indicate that this a maximal element with respect to $\succeq_i$, and not with respect to $\succeq'_i$. By assumption, $a^*\sim_i b$ implies $a^*\sim'_i b$, so $a^*\sim'_i b_i(A(r)|\succeq_i)$ for both i. By definition, $b_i(A(r)|\succeq_i)\succeq_i b$ for all $b\in A(r)$, so $a^*\succeq_i b$ for all $b\in A(r)$. Because $a^*\succeq_i b$ implies $a^*\succeq'_i b$, it follows that $a^*\succeq'_i b$ for all $b\in A(r)$. Together with $a^*\sim'_i b_i(A(r)|\succeq_i)$, we obtain that $b_i(A(r)|\succeq_i)\succeq'_i b$ for all $b\in A(r)$. We know therefore that $b_i(A(r)|\succeq_i)\sim'_i b_i(A(r)|\succeq'_i)$ for both i. Because $a^*\sim'_i b_i(A(r)|\succeq_i)$ for both i, we have that $a^*\sim'_i b_i(A(r)|\succeq'_i)$ for both i.

We show now that a^* is Pareto optimal in B^2. Suppose not. Then, there is some $b\in A$ with $a^*\preceq'_i b$ for both i and $a^*\prec'_i b$ for some i. By assumption on the preferences, it follows that $a^*\preceq_i b$ for both i and $a^*\prec_i b$ for some i, which is a contradiction since a^* is Pareto optimal in B^1. Hence, $a^*\sim'_i b_i(A(r)|\succeq'_i)$ for both i and a^* is Pareto optimal in B^2. This means that $a^*\in\varphi^{\mathcal{F}}(B^2)$. So, $\varphi^{\mathcal{F}}$ satisfies unchanged contour independence.

Now, suppose that φ is a solution on $\mathcal{B}^*$ satisfying Pareto optimality, essential uniqueness, $\mathcal{F}$–monotonicity and unchanged contour independence. We show that $\varphi=\varphi^{\mathcal{F}}$. Let $B=(A(t), e, \succeq_1, \succeq_2)\in\mathcal{B}^*$ with set of alternatives $A(t)\in\mathcal{F}$, and let $a^*\in\varphi^{\mathcal{F}}(B)$. Since both $\varphi^{\mathcal{F}}$ and φ are essentially unique, it suffices to show that $a^*\in\varphi(B)$. Suppose not. Then, because $\varphi(B)$ is essentially unique and nonempty, there is some $b^*\in\varphi(B)$ such that $a^*\nsim_i b^*$ for at least one agent i. Because $\varphi^{\mathcal{F}}$ and φ are Pareto optimal, we know that both a^* and b^* are Pareto optimal in B. Given that $a^*\nsim_i b^*$ for some agent i, it follows that $a^*\succ_i b^*$ for some agent i, and $a^*\prec_j b^*$ for the other agent j. Assume, without loss of generality, that $a^*\succ_1 b^*$ and $a^*\prec_2 b^*$. By definition of the solution $\varphi^{\mathcal{F}}$, there is some $r\in[0, t]$ such that $a^*\sim_i b_i(A(r))$ for both agents i. Hence, $b^*\prec_1 b_1(A(r))$ and $b^*\succ_2 b_2(A(r))$.

Let u_1, u_2 be an arbitrary utility representation for the preferences $\succeq_1$, $\succeq_2$. By assumption, the induced bargaining problem in utility space (S, d) satisfies the Pareto connectedness condition. Since a^*, b^* are Pareto optimal with $a^*\succ_1 b^*$ and $a^*\prec_2 b^*$, we then know that there is some Pareto optimal alternative $c\in A(t)$ with $a^*\succ_1 c\succ_1 b^*$ and $a^*\prec_2 c\prec_2 b^*$. Hence, $b_1(A(r))\succ_1 c\succ_1 b^*$ and $b_2(A(r))\prec_2 c\prec_2 b^*$.

We now define continuous utility functions u'_1, u'_2 on $A^{\mathcal{F}}$ by

$$u_1{}'(a) = \begin{cases} u_1(a), & \text{if } u_1(a)\le u_1(c) \\ u_1(c), & \text{if } u_1(a)>u_1(c) \end{cases},$$

$$u_2{}'(a) = \begin{cases} u_2(a), & \text{if } u_2(a)\ge u_2(c) \\ u_2(c), & \text{if } u_2(a)<u_2(c) \end{cases}.$$

Let $\succeq'_1$, $\succeq'_2$ be the preferences induced by u'_1, u'_2. Let the bargaining problem B' be given by $B'=(A(t), e, \succeq'_1, \succeq'_2)$. Let (S', d') be the induced bargaining problem in utility space. We show that (S', d') satisfies the Pareto connectedness condition. Let $NW_c=\{(x, y)\in\mathbb{R}^2|x\le u_1(c) \text{ and } y\ge u_2(c)\}$ be the set of utility pairs to the North-West of $(u_1(c), u_2(c))$. It may be verified easily that $S'=S\cap NW_c$, where S is the set of feasible utilities in the original bargaining problem B. See Fig. 5 for an illustration of this fact.

Moreover, because $(u_1(c), u_2(c))$ is a Pareto optimal utility pair in S, it follows that the set of Pareto optimal utility pairs in S' is given by $PO(S')=PO(S)\cap NW_c$, where $PO(S)$ is the set of Pareto optimal utility pairs in S. See Fig. 5 for an illustration. By assumption, the set $PO(S)$ is connected. Hence, the set $PO(S')=PO(S)\cap NW_c$ is also connected, which means that (S', d') satisfies the Pareto connectedness condition. Because $\succeq'_1$, $\succeq'_2$ are truncations of $\succeq_1$, $\succeq_2$, it follows, by condition (2) of $\mathcal{F}-$completeness of $\mathcal{B}^*$, that $B'\in\mathcal{B}^*$.

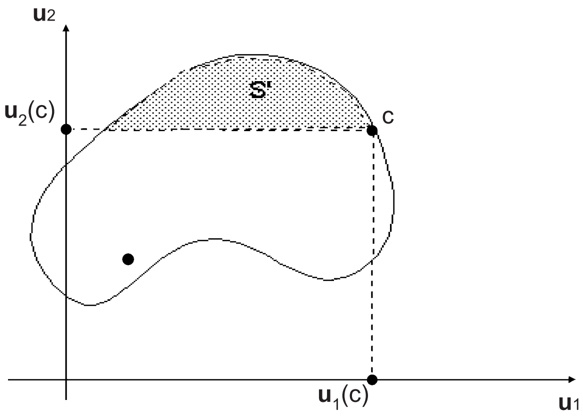

Fig. 5. (S' , d') satisfies Pareto connectedness condition.

Consider now the reduced bargaining problem $B''=(A(r), e, \succeq'_1, \succeq'_2)$, where r is as defined above. Because $c \succ_2 b_2(A(r))$ it follows that $a \prec_2 c$ for all $a \in A(r)$. By construction of u'_2, $u'_2(a)=u_2(c)$ for all $a \in A(r)$. Therefore, agent 2's utility in B'' is equal to $u_2(c)$ for all alternatives in $A(r)$, and hence is constant on $A(r)$. Because $c \prec_1 b_1(A(r))$ it follows that there is some $a \in A(r)$ with $u_1(a)>u_1(c)$. By construction of u'_1, it follows that in the bargaining problem B'' agent 1's maximum utility in $A(r)$ is $u_1(c)$. Hence, the bargaining problem B'' has a unique Pareto optimal utility pair, namely $(u_1(c), u_2(c))$. In particular, the set of Pareto optimal utility pairs in B'' is connected, and hence B'' satisfies the Pareto connectedness condition. Because $B' \in \mathcal{B}^*$, it follows, by condition (1) of $\mathcal{F}-$completeness, that $B'' \in \mathcal{B}^*$.

Let $a'' \in \varphi(B'')$. Since φ is Pareto optimal, a'' should be Pareto optimal in B'' and hence, $(u'_1(a''), u'_2(a''))=(u_1(c), u_2(c))$. We thus have that $u'_1(a'')=u_1(c)=u'_1(c)$, and hence, $a'' \sim'_1 c$.

Now, let $a' \in \varphi(B')$. Because B'' is a reduction of B' within the family $\mathcal{F}$, it follows by $\mathcal{F}-$monotonicity that $a' \succeq'_1 a'' \sim'_1 c$. By assumption, $c \succ_1 b^*$. By construction of u'_1, it follows that $c \succ'_1 b^*$. We may thus conclude that $a' \succ'_1 b^*$.

By construction of the utility functions u'_1, u'_2, it may be verified that for both agents i the lower contour set, upper contour set and indifference set with respect to b^* are the same in B' as in B. Because $b^* \in \varphi(B)$ and φ satisfies unchanged contour independence, it follows that $b^* \in \varphi(B')$. Above we have seen that every $a' \in \varphi(B')$ satisfies $a' \succ'_1 b^*$, and therefore, b^* cannot be in $\varphi(B')$, which is clearly a contradiction. We may thus conclude that $a^* \in \varphi(B)$, which implies that $\varphi(B)=\varphi^{\mathcal{F}}(B)$. □

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