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Incomplete information and iterated strict dominance

By Christian W. Bach[ab] and Andrés Perea[cd]

[a]Department of Economics, University of Liverpool Management School, Chatham Street, Liverpool, UK-L69 7ZH; e-mail: c.w.bach@liverpool.ac.uk
[b]EPICENTER, School of Business and Economics, Maastricht University, NL-6200, MD, Maastricht
[c]Department of Quantitative Economics, School of Business and Economics, Maastricht University, NL-6200, MD, Maastricht; e-mail: a.perea@maastrichtuniversity.nl
[d]EPICENTER, School of Business and Economics, Maastricht University, NL-6200, MD, Maastricht

Abstract

The solution concept of iterated strict dominance for static games with complete information recursively deletes choices that are inferior. Here, we devise such an algorithm for the more general case of incomplete information. The ensuing solution concept of generalized iterated strict dominance is characterized in terms of common belief in rationality as well as in terms of best response sets. Besides, we provide doxastic conditions that are necessary and sufficient for modelling complete information from a one-person perspective.

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1. Introduction

The most basic game-theoretic model of an interactive decision situation is a static game with complete information. Accordingly, a set of players with a choice set for every player is given as well as the payoff structure of the game defined by specifying for every player a unique utility function that maps choice combinations to payoff values. Such a model describes the essential features of an interactive situation. In game theory different solution concepts are then proposed which identify for every player some choices—in line with a reasonability criterion or decision rule—as the solution of the game.

According to a fundamental idea in game theory a choice that fares worse than some other choice or some randomization over choices against every possible combination of opponents' choices is called strictly dominated and is deemed to be an unreasonable option for the corresponding player. The widespread solution concept of iterated strict dominance builds on this idea. In a first round all strictly dominated choices are eliminated for every player. The ensuing reduced game is then considered and in a second round all strictly dominated choices are eliminated for every player therein. It is continued in this fashion until no more strict dominance relations can be identified. The surviving choices for each player of this algorithm form the solution of the game. For finite games iterated strict dominance exhibits the convenient properties of stopping after finitely many rounds, resulting in a non-empty output, and being order-independent. In fact, historically the idea of iteratively eliminating strictly dominated choices can be traced back to the early days of game theory (e.g. Nash, 1951, pp. 292–93).

In static games with complete information players do not face any uncertainty about the payoff structure. All utility functions are commonly known among the players. However, in many interactive decision situations in the real world this assumption is not satisfied. For instance, a firm does typically not know the cost structure and thus the profit function of a competitor or a participant in an auction is usually not certain about the valuation of the other participants. It is thus relevant to explore strategic decision situations involving payoff uncertainty too. The corresponding game-theoretic framework is provided by static games with incomplete information. A direct way to accommodate payoff uncertainty simply specifies a set of—rather than unique—utility functions for every player. Complete information can thus be viewed as the special case of incomplete information, where all sets of utility functions are singletons.

The analysis of incomplete information has been pioneered by Harsanyi (1967–68). He models payoff uncertainty by the notion of a type and proposes the solution concept of Bayesian equilibrium. Intuitively, his solution concept embeds a best response property in a type structure that determines the belief hierarchies on the players' utility functions based on a common prior. In relation to the special case of complete information, Bayesian equilibrium can actually be shown to constitute the incomplete information counterpart to correlated equilibrium (cf. Battigalli and Siniscalchi, 2003a; Bach and Perea, 2017).

While Bayesian equilibrium has become the most prevalent solution concept for incomplete information games, more recently, the idea of rationalizability—due to Bernheim (1984) and Pearce (1984)—has been generalized to incomplete information games. In particular, the solution concepts of weak and strong Δ-rationalizability have been introduced by Battigalli (2003), and further analysed by Battigalli and Siniscalchi (2003a, 2007), Battigalli *et al.* (2011), Battigalli and Prestipino (2013), as well as Dekel and Siniscalchi (2015). Intuitively, Δ-rationalizability concepts iteratively delete choice utility pairs by some best response requirement and allow for exogenous restrictions on the first-order beliefs. Δ-rationalizability has been applied to auctions by Battigalli and Siniscalchi (2003b), to signalling games by Battigalli (2006), as well as to static implementation by Ollar and Penta (2017). A backward inductive variant of rationalizability for dynamic games with incomplete information has been proposed by Penta (2017) and applied to dynamic implementation by Penta (2015). Yet other incomplete information generalizations of rationalizability are given by Ely and Pęski (2006)'s interim independent rationalizability as well as by Dekel *et al.*'s (2007) interim correlated rationalizability. While the former solution concept in its iterative procedure requires a player's belief about the opponents' choices conditional on the opponents' types to be independent, the latter solution concept does not impose any independence restriction. In contrast to Δ-rationalizability the two incomplete information notions of interim rationalizability fix the belief hierarchies on utilities. This constitutes the essential difference between interim independent rationalizability and interim correlated rationalizability on the one hand and Δ-rationalizability on the other hand.

Actually, Battigalli and Siniscalchi (1999) as well as Battigalli (2003) indicate that Δ-rationalizability notions are equivalent to iterated strict dominance procedures for the class of static games. A characterization of Δ-rationalizability in terms of an iterated elimination procedure based on the notion of Δ-dominance is given by Cappelletti (2010) and for the special case of no exogenous belief restrictions in terms of so-called interim iterated dominance by Battigalli *et al.* (2011). A similar iterated elimination procedure has also been formulated and used in the context of mechanism design by Bergemann and Morris (2003).

Here, we propose a simple solution concept for incomplete information games called *generalized iterated strict dominance* as a direct analogue to the complete information solution concept of iterated strict dominance. Intuitively, a game is expressed from a one-person perspective in terms of decision problems which are then iteratively reduced by some strict dominance requirement and the resulting output yields choice utility function pairs for every player. Our solution concept as well as the incomplete information framework is kept entirely non-doxastic. Neither types nor beliefs appear in any form. In this sense, our approach is basic and as sparse as possible. Doxastic notions only appear in the reasoning realm based on epistemic models for games. A clear dichotomy between the classical sphere—game model as well as solution concept—and the epistemic sphere—epistemic model and reasoning concept—thus ensues. Moreover, we epistemically characterize generalized iterated strict dominance in terms of common belief in rationality and also give a characterization in terms of best response sets. Besides, we provide doxastic correctness conditions on belief hierarchies, within the mind of a single reasoner, that are necessary and sufficient for modelling the special case of complete information.

Compared to Δ-rationalizability, generalized iterated strict dominance does not invoke any best response requirements or beliefs whatsoever. By merely using strict dominance arguments our solution concept constitutes a very elementary and practical tool for the class of games with payoff uncertainty. In terms of output generalized iterated strict dominance coincides with Δ-rationalizability, if no exogenous belief restrictions are admitted. Also, without such doxastic restrictions, generalized iterated strict dominance becomes essentially equivalent to some particular way of iterating Δ-dominance. In terms of formulation generalized iterated strict dominance—by using the notion of decision problem and by being constructed in a type-free incomplete information framework—differs from the Δ-rationalizability and Δ-dominance concepts.

The pioneering work on incomplete information by Harsanyi (1967–68) is based on a one-person perspective. Accordingly, the strategic situation is analysed entirely from the viewpoint of a single player. For instance, as Harsanyi (1967–68, p. 170) writes it is some *[…] player j (from whose point of view we are analyzing the game) […]*, and Harsanyi (1967–68, p. 175) states that *[…] we are interested only in the decision rules that player j himself will follow […]*. Philosophically, a one-person perspective approach treats game theory as an interactive extension of decision theory. While game theoretic notions are of course inherently interactive, one-person perspective modelling formalizes solution concepts or reasoning patterns entirely within the mind of a single player. Such an approach departs from the standard way game theory proceeds, which simultaneously imposes conditions on the beliefs and actions of all players. The typical multi-player modelling results in the notion of state which fixes for every player his actual choice as well as his actual beliefs. In contrast, a one-person perspective *modus operandi* utterly dispenses with states as only the actual beliefs of a single player are modelled—doxastic conditions concerning the

$\Gamma_{Alice}(u_{Alice})$

	blue	red	yellow
blue	0	3	3
red	2	0	2
yellow	1	1	0

$\Gamma_{Bob}(u_{Bob})$

	blue	red	yellow
blue	0	1	1
red	3	0	3
yellow	2	2	0

$\Gamma_{Bob}(u'_{Bob})$

	blue	red	yellow
blue	0	2	2
red	1	0	1
yellow	3	3	0

Fig. 1. One-person perspective representation in terms of decision problems

opponents only enter as conditions on the actual higher-order beliefs of one player. Intuitively, a person involved in an interactive choice situation deliberates about the thinking of his opponents and their possible choices. All of these interactive cognitive processes occur solely in his mind. Philosophically, a one-person perspective approach thus appears to be a rather natural way of conducting game theory.

Inspired by Harsanyi (1967–68) and the above philosophical considerations we take a one-person perspective approach here. Notably, we construct the solution concept of generalized iterated strict dominance based on a one-person perspective representation of a game in terms of decision problems. In the reasoning realm our definition of the pivotal notion of rational choice under common belief in rationality in the context of payoff uncertainty merely imposes conditions on the reasoner himself. Also, our characterization of the special case of payoff certainty only restricts the thinking of a single player.

We keep our formal framework as basic and accessible as possible. In particular, games with incomplete information are defined in a minimal way without invoking any types and are thus belief-free. Doxastic notions are left entirely for the reasoning realm of epistemic models. The intended simplicity and practicability of our framework and solution concept is supposed to facilitate and encourage the use of generalized iterated strict dominance for applications in economics or beyond such as management or political theory. For instance, in pricing games firms may have no information about their competitors' characteristics such as their cost structures. Furthermore, in auctions participants can be uncertain about each other's valuations, which is indeed typically assumed in public auctions or Internet auctions. More generally, incomplete information settings of mechanism design or implementation could be analysed with this non-equilibrium solution concept.

The idea of generalized iterated strict dominance is now illustrated by means of an example. Suppose that Alice as well as Bob are both attending a party and have to decide what colour to wear. Their wardrobes are similar in the sense that only garments of three colours can be found inside: blue, red, and yellow. While Alice prefers blue to red to yellow, she cannot remember the precise colour preferences of Bob. Alice merely recalls that he either prefers red to yellow to blue or yellow to blue to red. Both players dislike most wearing a garment of equal colour at the party. This interactive decision situation—or game—is represented from a one-person perspective by the decision problem $\Gamma_{Alice}(u_{Alice})$ for Alice and the two decision problems $\Gamma_{Bob}(u_{Bob})$ and $\Gamma_{Bob}(u'_{Bob})$ for Bob in Fig. 1, where a decision problem contains choices of the respective player, choices of his opponent, as well as payoffs for choice combinations.

In its first round generalized iterated strict dominance searches for strict dominance relations for each of the decision problems. Given Alice's utility function u_{Alice}, her choice yellow is strictly dominated by the randomized choice that assigns probability 0.4 to blue and 0.6 to red, and hence yellow is eliminated in Alice's decision problem $\Gamma_{Alice}(u_{Alice})$. Given Bob's utility function u_{Bob}, his choice blue is strictly dominated by the randomized choice that assigns probability 0.4 to red and 0.6 to yellow, and thus blue is deleted in Bob's decision problem $\Gamma_{Bob}(u_{Bob})$. Given Bob's utility function u'_{Bob}, his choice red is strictly dominated by the randomized choice that assigns probability 0.6 to blue and 0.4 to yellow, and hence red is eliminated in Bob's decision problem $\Gamma_{Bob}(u'_{Bob})$. Now, for all decision problems of Alice—in fact there merely exists a single one—yellow has been eliminated, and is consequently also deleted in both of Bob's decision problems. For Bob there exists no choice that has been eliminated for all of his decision problems, and hence all of his choices are kept in Alice's decision problem. In the second round of the algorithm, for Bob's utility function u'_{Bob}, his choice blue is strictly dominated by yellow against Alice's reduced choice set consisting of blue and red only. Thus, blue is deleted in Bob's decision problem $\Gamma_{Bob}(u'_{Bob})$. Since blue has already been identified as a strictly dominated choice for u_{Bob} in the preceding step, it is the case that for all decision problems of Bob blue is a strictly dominated choice and is hence also eliminated in Alice's decision problem $\Gamma_{Alice}(u_{Alice})$. However, in the third round red then emerges as a strictly dominated choice for Alice given her utility function u_{Alice}, since it is strictly dominated by blue against Bob's reduced choice set consisting of red and yellow only, and blue remains as her unique choice in her decision problem $\Gamma_{Alice}(u_{Alice})$. Consequently, in the fourth round, given Bob's utility function u_{Bob} his choice yellow becomes a strictly dominated choice, as it is strictly dominated by red against Alice's reduced choice set consisting of blue only. It is thus the case that in decision problem $\Gamma_{Bob}(u_{Bob})$ for Bob's utility function u_{Bob} the choice red survives, and in his decision problem $\Gamma_{Bob}(u'_{Bob})$ for his utility function u'_{Bob} the choice yellow remains. Therefore, generalized iterated strict dominance yields $\{(blue, u_{Alice})\} \times \{(red, u_{Bob}), (yellow, u'_{Bob})\}$ as the solution of the game. In other words, Alice will choose a blue dress for the party, while Bob will appear in a red suit, if he prefers red to yellow to blue, and in a yellow suit, if he prefers yellow to blue to red.

We proceed as follows. First of all, in Section 2, the formal framework is laid out and some basic notation fixed. In particular, incomplete information as well as common belief in rationality are defined and illustrated by means of an example. Then, in Section 3, the solution concept of generalized iterated strict dominance is constructed as an algorithm on decision problems using strict dominance arguments. Roughly speaking, for a given player a decision problem is formed for each of his payoff structures in every round, with all opponents' choices being deleted that are strictly dominated in every decision problem of the respective opponent in the previous round, and subsequently the player's choices that are then strictly dominated are eliminated. An example illustrates the application of generalized iterated strict dominance to specific games. Moreover, in Section 4, two characterizations of our solution concept are provided. In terms of reasoning the output of generalized iterated strict dominance is shown to be equivalent to rational choice under common belief in rationality (Theorem 1). Also, a characterization of generalized iterated strict dominance—without recourse to any iterative procedure—is given by means of best response sets (Theorem 2). Finally, in Section 5, the special case of complete information is considered. A characterization of complete information is given in terms of doxastic correctness conditions imposed on a single player's reasoning (Theorem 3). Thereby a purely doxastic foundation is provided for payoff uncertainty from a one-person perspective. Besides, our solution concept is shown to coincide with iterated strict dominance for static games with complete information in terms of output.

2. Preliminaries

A *static game with incomplete information* can be formally represented by a tuple:

$$\Gamma = (I, (C_i)_{i\in I}, (U_i)_{i\in I})$$

where I denotes a finite set of players, C_i denotes player i's finite choice set, and U_i denotes the finite set of player i's possible utility functions. Every utility function $u_i \in U_i$ is of the form: $u_i : \times_{j\in I} C_j \to \mathbb{R}$.

In order to express beliefs and interactive beliefs about choices and utility functions an epistemic structure needs to be added to the game. Formally, let $\Gamma = (I, (C_i)_{i\in I}, U_i)_{i\in I})$ be a static game with incomplete information. An *epistemic model* of Γ is a tuple:

$$\mathcal{M}^\Gamma = ((T_i)_{i\in I}, (b_i)_{i\in I})$$

where T_i is a finite set of types, and $b_i : T_i \to \Delta(C_{-i} \times T_{-i} \times U_{-i})$ assigns to every type $t_i \in T_i$ a probability measure $b_i[t_i]$ on the set of opponents' choice type utility function combinations. Note that for every type an infinite belief hierarchy about the respective opponents' choices and utility functions can be derived. Also, marginal beliefs can be inferred from a type. For instance, every type $t_i \in T_i$ induces a belief on the opponents' choice combinations by marginalizing the probability measure $b_i[t_i]$ on the space C_{-i}. For simplicity sake, no additional notation is introduced for marginal beliefs. In the sequel, it should always be clear from the context which belief $b_i[t_i]$ refers to.

In our epistemic approach payoff uncertainty is treated symmetrically to strategic uncertainty. As the latter concerns the respective opponents' choices, the former is also defined in terms of the probability measures in the epistemic models with respect to the respective *opponents*' utility functions only. A player's own choices and utility functions enter his reasoning exclusively via higher-order beliefs. In particular, players thus entertain no uncertainty about their own utility functions in our epistemic approach. This treatment is in line with Harsanyi (1967–68), who also assumes that each player knows his own utility function, and more generally, that the uncertainty concerns the opponents of the player from whose point of view the game is analysed.[1] However, the special case of players being uncertain about their *own* payoffs could be accommodated by extending the space of uncertainty for every player $i \in I$ from $C_{-i} \times T_{-i} \times U_{-i}$ to $C_{-i} \times T_{-i} \times (\times_{j\in I} U_j)$. Alternatively, a reasoner's actual utility function could be defined as the expectation over the set U_i. This modelling choice does not affect the subsequent results. In our treatment, a type only specifies the epistemic mental state of a player, not his utility function. In this sense we follow Harsanyi's (1967–68) approach, which separates the utility component from the epistemic component.[2]

Some further notions and notation are now introduced. For that purpose consider a game Γ, an epistemic model $\mathcal{M}^\Gamma$ of it, and fix two players $i, j \in I$ such that $i \neq j$. A type $t_i \in T_i$ of i is said to *deem possible* some choice type utility function combination (c_{-i}, t_{-i}, u_{-i})

1 Cf. Harsanyi (1967–68), p. 163 and p. 170.

2 Cf. Harsanyi (1967–68), pp. 169–171.

of his opponents, if $b_i[t_i]$ assigns positive probability to (c_{-i}, t_{-i}, u_{-i}). Analogously, t_i deems possible some type t_j of his opponent, if $b_i[t_i]$ assigns positive probability to t_j. For each choice-type-utility function combination (c_i, t_i, u_i), the *expected utility* is given by $v_i(c_i, t_i, u_i) = \sum_{c_{-i} \in C_{-i}} b_i[t_i](c_{-i}) \cdot u_i(c_i, c_{-i})$. A choice $c_i \in C_i$ is said to be *optimal* for the type utility function pair (t_i, u_i), if $v_i(c_i, t_i, u_i) \geq v_i(c_i', t_i, u_i)$ for all $c_i' \in C_i$. Moreover, a type $t_i \in T_i$ is said to *believe in the opponents' rationality*, if t_i only deems possible choice type utility function combinations (c_{-i}, t_{-i}, u_{-i}) such that c_j is optimal for (t_j, u_j) for every opponent $j \in I \setminus \{i\}$. Interactive belief in rationality with payoff uncertainty can then be defined by iterating belief in rationality. A type $t_i \in T_i$ expresses *1-fold belief in rationality*, if t_i believes in the opponents' rationality, and *k-fold belief in rationality* for some $k > 1$, if t_i only assigns positive probability to types $t_j \in T_j$ for all $j \in I \setminus \{i\}$ such that t_j expresses $(k-1)$-fold belief in rationality. Common belief in rationality then ensues as interactive belief in rationality throughout the reasoner's belief hierarchy. Formally, a type $t_i \in T_i$ expresses *common belief in rationality*, if t_i expresses k-fold belief in rationality for all $k \geq 1$.

In a game a player reasons about his opponents as well as the game and then makes a choice. While reasoning patterns can be modelled as conditions on belief hierarchies, a decision rule connects the reasoning with a choice. The basic decision rule of rational choice under common belief in rationality can be defined in the context of payoff uncertainty as follows.

Definition 1 Let $\Gamma = (I, (C_i)_{i \in I}, (U_i)_{i \in I})$ be a static game with incomplete information, $i \in I$ some player, and $u_i \in U_i$ some utility function of player i. A choice $c_i \in C_i$ of player i is rational for utility function u_i under common belief in rationality, if there exists an epistemic model $\mathcal{M}^\Gamma$ of Γ with a type $t_i \in T_i$ of player i such that c_i is optimal for (t_i, u_i) and t_i expresses common belief in rationality.

Note that the decision rule of rational choice for some utility function under common belief in rationality is formulated here from a one-person perspective in Definition 1. Indeed, conditions are exclusively imposed on the reasoner himself: his mind, i.e. his type (or equivalently his implicit belief hierarchy), his preferences, and his choice. No conditions on other players' actual thinking or choices are invoked.

The epistemic notion of common belief in rationality for incomplete information games has been formalized and employed in different forms for epistemic foundations of the Δ-rationalizability variants by Battigalli and Siniscalchi (1999, 2002, 2007), Battigalli *et al.* (2011), as well as Battigalli and Prestipino (2013). Besides, Battigalli *et al.* (2011) also give an epistemic foundation of interim correlated rationalizability in terms of common belief in rationality.

An illustration of the concept of common belief in rationality is provided by the following example.

Example 1 Consider the static game with incomplete information between *Alice* and *Bob* depicted in Fig. 2. Let the utility functions of *Alice* represented in the first and second matrices of Fig. 2 be denoted by u_A and the ones represented by the third and fourth matrices by u_A'. Similarly, let the utility functions of *Bob* represented in the first and third matrices in Fig. 2 be denoted by u_B and the ones represented by the second and fourth matrices by u_B'.

Alice \ Bob	d	e	f
a	3, 3	2, 2	1, 0
b	2, 2	1, 1	3, 0
c	0, 1	0, 3	0, 0

Alice \ Bob	d	e	f
a	3, 1	2, 2	1, 0
b	2, 3	1, 1	3, 0
c	0, 1	0, 1	0, 0

Alice \ Bob	d	e	f
a	1, 3	3, 2	1, 0
b	2, 2	1, 1	1, 0
c	0, 1	0, 3	0, 0

Alice \ Bob	d	e	f
a	1, 1	3, 2	1, 0
b	2, 3	1, 1	1, 0
c	0, 1	0, 1	0, 0

Fig. 2. A two player static game with incomplete information

Suppose the epistemic model $\mathcal{M}^\Gamma$ of Γ given by the sets of types $T_{Alice} = \{t_A, t'_A\}$, $T_{Bob} = \{t_B, t'_B\}$, and the following induced belief functions

– $b_{Alice}[t_A] = (e, t_B, u_B)$,
– $b_{Alice}[t'_A] = (d, t'_B, u'_B)$,
– $b_{Bob}[t_B] = (a, t_A, u_A)$,
– $b_{Bob}[t'_B] = \frac{1}{2}\Big(a, t'_A, u_A) + \frac{1}{2}(b, t'_A, u'_A)$.

Accordingly, type t_A assigns probability 1 to the choice type utility function combination (e, t_B, u_B). Analogously, the induced beliefs of types t'_A and t_B are obtained. *Bob*'s type t'_B assigns probability $\frac{1}{2}$ to the choice type utility function combination (a, t'_A, u_A) and probability $\frac{1}{2}$ to the choice type utility function combination (b, t'_A, u'_A). Note that *Alice*'s type t_A does not believe in *Bob*'s rationality, as *e* is not optimal for the type utility function pair (t_B, u_B) she believes him to be characterized by. In particular, it follows that t_A does not express common belief in rationality. However, *Alice*'s type t'_A expresses common belief in rationality. Indeed, t'_A believes in *Bob*'s rationality, as *d* is optimal for *Bob*'s type utility function pair (t'_B, u'_B). Also, t'_B believes in *Alice*'s rationality, since *a* is optimal for *Alice*'s type utility function pair (t'_A, u_A) and *b* is optimal for *Alice*'s type utility function pair (t'_A, u'_A). As t'_A only deems possible *Bob*'s type t'_B, and t'_B only deems possible *Alice*'s type t'_A, it follows inductively that t'_A expresses common belief in rationality. Hence, *a* is rational for u_A under common belief in rationality, *b* is rational for u'_A under common belief in rationality, and *d* is rational for u'_B under common belief in rationality. ♣

3. Generalized iterated strict dominance

Games can be expressed from a one-person perspective based on the notion of decision problems. Formally, given a game $\Gamma = (I, (C_i)_{i \in I}, (U_i)_{i \in I})$, a player $i \in I$, and a utility function $u_i \in U_i$, a decision problem $\Gamma_i(u_i) = (D_i, D_{-i}, u_i)$ for player *i* consists of choices $D_i \subseteq C_i$ for *i*, choice combinations $D_{-i} \subseteq C_{-i}$ for *i*'s opponents, as well as the utility function u_i restricted to $D_i \times D_{-i}$. The special case of $\Gamma_i(u_i) = (C_i, C_{-i}, u_i)$ is called a full decision problem of player *i*. A game can then be expressed as a set of full decision problems for every player, and the tuple $(\cup_{u_i \in U_i}\{(C_i, C_{-i}, u_i)\})_{i \in I}$ constitutes the *one-person perspective form* of Γ.

In decision problems, choice rules such as strict dominance can be formally defined. Indeed, given a utility function $u_i \in U_i$ for player *i* and his corresponding decision problem $\Gamma_i(u_i) = (D_i, D_{-i}, u_i)$, a choice $c_i \in D_i$ is called strictly dominated, if there exists a probability measure $r_i \in \Delta(D_i)$ such that $u_i(c_i, c_{-i}) < \sum_{c'_i \in D_i} r_i(c'_i) \cdot u_i(c'_i, c_{-i})$ for all $c_{-i} \in D_{-i}$.

With the notions of decision problem and strict dominance on decision problems, the solution concept of *generalized iterated strict dominance* is defined as follows.

Definition 2 Let $\Gamma = (I, (C_i)_{i \in I}, (U_i)_{i \in I})$ be a static game with incomplete information and $(\cup_{u_i \in U_i}\{(C_i, C_{-i}, u_i)\})_{i \in I}$ the one-person perspective form of Γ.

Round 1 For every player $i \in I$ and for every utility function $u_i \in U_i$ consider the initial decision problem $\Gamma_i^0(u_i) := (C_i^0(u_i), C_{-i}^0(u_i), u_i)$, where $C_i^0(u_i) := C_i$ and $C_{-i}^0(u_i) := C_{-i}$.

Step 1.1 Set $C_{-i}^1(u_i) := C_{-i}^0(u_i)$.
Step 1.2 Form $\Gamma_i^1(u_i) := (C_i^1(u_i), C_{-i}^1(u_i), u_i)$, where $C_i^1(u_i) \subseteq C_i^0(u_i)$ only contains choices $c_i \in C_i$ for player i that are not strictly dominated in the decision problem $(C_i^0(u_i), C_{-i}^1(u_i), u_i)$.

Round $k > 1$ For every player $i \in I$ and for every utility function $u_i \in U_i$ *consider the reduced decision problem* $\Gamma_i^{k-1}(u_i) := (C_i^{k-1}(u_i), C_{-i}^{k-1}(u_i), u_i)$.

Step k.1 Form $C_{-i}^k(u_i) \subseteq C_{-i}^{k-1}(u_i)$ by eliminating from $C_{-i}^{k-1}(u_i)$ every opponents' choice combination $c_{-i} \in C_{-i}^{k-1}(u_i)$ that contains for some opponent $j \in I \setminus \{i\}$ a choice $c_j \in C_j$ for which there exists no utility function $u_j \in U_j$ such that $c_j \in C_j^{k-1}(u_j)$.
Step k.2 Form $\Gamma_i^k(u_i) := (C_i^k(u_i), C_{-i}^k(u_i), u_i)$, where $C_i^k(u_i) \subseteq C_i^{k-1}(u_i)$ only contains choices $c_i \in C_i^{k-1}(u_i)$ for player i that are not strictly dominated in the decision problem $(C_i^{k-1}(u_i), C_{-i}^k(u_i), u_i)$.

The set $GISD := \times_{i \in I} GISD_i \subseteq \times_{i \in I}(C_i \times U_i)$ constitutes the output of generalized iterated strict dominance, where for every player $i \in I$ the set $GISD_i \subseteq C_i \times U_i$ only contains choice utility function pairs $(c_i, u_i) \in C_i \times U_i$ such that $c_i \in C_i^k(u_i)$ for all $k \geq 0$.

The algorithm is initiated from the one-person perspective form of the game and iteratively eliminates strictly dominated choices from decision problems for all players. In every round a decision problem for a player is formed by first eliminating all opponents' choices that are strictly dominated in every decision problem for that opponent in the previous round, and subsequently eliminating the player's choices that are strictly dominated. In fact, for every player the algorithm yields a set of choice utility function pairs as output. Due to the presence of incomplete information the algorithm thus identifies choices relative to payoffs.

The following remark draws attention to some useful properties of the generalized iterated strict dominance algorithm.

Remark 1 Let $\Gamma = (I, (C_i)_{i \in I}, (U_i)_{i \in I})$ be a static game with incomplete information. The solution concept of generalized iterated strict dominance yields a non-empty output, i.e. $GISD \neq \emptyset$, is finite, i.e. there exists $n \in \mathbb{N}$ such that $\Gamma_i^k(u_i) = \Gamma_i^n(u_i)$ for all $k \geq n$, for all utility functions $u_i \in U_i$, and for all players $i \in I$, as well as qualifies as order-independent, i.e. the final output of generalized iterated strict dominance does not depend on the specific order of elimination.

The non-emptiness of the algorithm follows from the fact that at no round it is possible to delete all choices for a given player by definition of strict dominance. As there are only finitely many choices for every player, the algorithm stops after finitely many rounds. As a choice remains strictly dominated if a decision problem is reduced, the order of elimination does not affect the eventual output of the algorithm.

Finally, generalized iterated strict dominance is illustrated by applying the algorithm to the two player game introduced in Example 1.

Example 2 Consider again the two player static game with incomplete information from Example 1. In order to apply generalized iterated strict dominance the game is first expressed in its one-person perspective form. Accordingly, a decision problem for every player and for each of the respective utility functions is formed in Fig. 3, where the choices

$\Gamma_A^0(u_A)$

	d	e	f
a	3	2	1
b	2	1	3
c	0	0	0

$\Gamma_A^0(u'_A)$

	d	e	f
a	1	3	1
b	2	1	1
c	0	0	0

$\Gamma_B^0(u_B)$

	a	b	c
d	3	2	1
e	2	1	3
f	0	0	0

$\Gamma_B^0(u'_B)$

	a	b	c
d	1	3	1
e	2	1	1
f	0	0	0

Fig. 3. Initial decision problems for *Alice* and *Bob*

$\Gamma_A^1(u_A)$

	d	e	f
a	3	2	1
b	2	1	3

$\Gamma_A^1(u'_A)$

	d	e	f
a	1	3	1
b	2	1	1

$\Gamma_B^1(u_B)$

	a	b	c
d	3	2	1
e	2	1	3

$\Gamma_B^1(u'_B)$

	a	b	c
d	1	3	1
e	2	1	1

Fig. 4. 1-fold reduced decision problems for *Alice* and *Bob*

$\Gamma_A^2(u_A)$

	d	e
a	3	2

$\Gamma_A^2(u'_A)$

	d	e
a	1	3
b	2	1

$\Gamma_B^2(u_B)$

	a	b
d	3	2

$\Gamma_B^2(u'_B)$

	a	b
d	1	3
e	2	1

Fig. 5. 2-fold reduced decision problems for *Alice* and *Bob*

of the respective decision-making player are represented as rows and the opponent's choices as columns.

In both $\Gamma_A^0(u_A)$ and $\Gamma_A^0(u'_A)$ the choice c is strictly dominated by b. For Bob the choice f is strictly dominated by e in his decision problems $\Gamma_B^0(u_B)$ and $\Gamma_B^0(u'_B)$. There are no further choices that can be ruled out for Alice or Bob with strict dominance given either of their utility functions. The 1-fold reduced decision problems Γ_A^1 and Γ_B^1 result as in Fig. 4.

In both $\Gamma_A^1(u_A)$ and $\Gamma_A^1(u'_A)$ those choices of Bob are eliminated that are strictly dominated in all initial decision problems Γ_B^0 for Bob, i.e. choice f. Then, the choice b can be deleted for Alice given u_A as it is strictly dominated by a in $(\{a,b\},\{d,e\},u_A)$, but not given u'_A as it is not strictly dominated in $(\{a,b\},\{d,e\},u'_A)$. Moreover, in both $\Gamma_B^1(u_B)$ and $\Gamma_B^1(u'_B)$ those choices of Alice are eliminated that are strictly dominated in all initial decision problems Γ_A^0 for Alice, i.e. choice c. Then, the choice e can be deleted for Bob given u_B as it is strictly dominated by d in $(\{d,e\},\{a,b\},u_B)$, but not given u'_B as it is not strictly dominated in $(\{d,e\},\{a,b\},u'_B)$. The 2-fold reduced decision problems Γ_A^2 and Γ_B^2 result as in Fig. 5.

Since there are no strict dominance relations in any of the 2-fold reduced decision problems Γ_A^2 and Γ_B^2, the algorithm stops and returns the set $GISD = GISD_{Alice} \times GISD_{Bob} = \{(a,u_A),(a,u'_A),(b,u'_A)\} \times \{(d,u_B),(d,u'_B),(e,u'_B)\}$ as a solution to this two player game with incomplete information. ♣

4. Characterization

A fundamental result in game theory—so-called Pearce's Lemma—due to Pearce (1984, Lemma 3) connects strict dominance and rationality. Accordingly, a choice in a two-player static game with complete information is strictly dominated, if and only if, it is irrational, i.e. not optimal for any belief about the opponent's choices. Formally, a choice $c_i \in C_i$ of some player $i \in I$ is called optimal for a belief $p \in \Delta(C_{-i})$ about the opponents' choices, if $\sum_{c_{-i}\in C_{-i}} p(c_{-i}) \cdot u_i(c_i,c_{-i}) \geq \sum_{c_{-i}\in C_{-i}} p(c_{-i}) \cdot u_i(c'_i,c_{-i})$ for all $c'_i \in C_i$. Similarly, in a game with incomplete information, a choice $c_i \in C_i$ is said to be optimal for a belief utility function pair $(p_i,\ u_i)$, where $p_i \in \Delta(C_{-i})$ and $u_i \in U_i$, if $\sum_{c_{-i}\in C_{-i}} p_i(c_{-i}) \cdot u_i(c_i, c_{-i}) \geq \sum_{c_{-i}\in C_{-i}} p_i(c_{-i}) \cdot u_i(c_i', c_{-i})$ for all $c_i' \in C_i$.

A slight generalization of Pearce's Lemma to finite incomplete information games in one-person perspective form is given by the following result.

Lemma 1 Let $\Gamma = (I, (C_i)_{i\in I}, (U_i)_{i\in I})$ be a static game with incomplete information, $(\cup_{u_i\in U_i}\{(C_i, C_{-i}, u_i)\})_{i\in I}$ the one-person perspective form of Γ, $i \in I$ some player, $u_i \in U_i$ some utility function of player i, and $\Gamma_i(u_i) = (D_i, D_{-i}, u_i)$ some decision problem of player i. A choice $c_i \in D_i$ is strictly dominated in $\Gamma_i(u_i)$, if and only if, there exists no probability measure $p \in \Delta(D_{-i})$ such that c_i is optimal for (p, u_i) in $\Gamma_i(u_i)$.

Proof. Consider the game $\Gamma' = (\{i, j\}, \{D_i', D_j'\}, \{u_i', u_j'\})$, where $D_i' = D_i$, $D_j' = \{d_j^{d_{-i}} : d_{-i} \in D_{-i}\}$, $u_i'(d_i, d_j^{d_{-i}}) = u_i(d_i, d_{-i})$ for all $d_i \in D_i'$ and for all $d_j^{d_{-i}} \in D_j'$, as well as $u_j'(d_i, d_j^{d_{-i}}) = 0$ for all $d_i \in D_i'$ and for all $d_j^{d_{-i}} \in D_j'$. Note that a choice $c_i \in D_i$ is strictly dominated in the decision problem $\Gamma_i(u_i)$, if and only if, it is strictly dominated in the two person game Γ'. By Pearce's Lemma applied to Γ', it then follows that c_i is strictly dominated in $\Gamma_i(u_i)$, if and only if, there exists no probability measure $p_i \in \Delta(D_{-i})$ such that c_i is optimal for (p_i, u_i) in $\Gamma_i(u_i)$. ■

Equipped with the generalized version of Pearce's Lemma the solution concept of generalized iterated strict dominance can be epistemically characterized by common belief in rationality for static games with incomplete information.

Theorem 1 Let $\Gamma = (I, (C_i)_{i\in I}, (U_i)_{i\in I})$ be a static game with incomplete information, $i \in I$ some player, $c_i \in C_i$ some choice for player i, and $u_i \in U_i$ some utility function of player i. The choice c_i is rational for u_i under common belief in rationality, if and only if, $(c_i, u_i) \in GISD_i$.

Proof. For the *only if* direction of the theorem define a set $(C_i \times U_i)^{CBR} := \{(c_i, u_i) \in C_i \times U_i : c_i$ is rational for u_i under common belief in rationality$\}$ for every player $i \in I$. It is shown, by induction on $k \geq 0$, that for every player $i \in I$ and for every choice utility function pair $(c_i, u_i) \in (C_i \times U_i)^{CBR}$, it is the case that $c_i \in C_i^k(u_i)$. Note that $c_i \in C_i^0(u_i)$ directly holds for all $(c_i, u_i) \in (C_i \times U_i)^{CBR}$ and for all $i \in I$, as $C_i^0(u_i) = C_i$ for all $u_i \in U_i$ and for all $i \in I$. Now consider some $k \geq 0$ and suppose that $c_i \in C_i^k(u_i)$ holds for every player $i \in I$ and for every choice utility function pair $(c_i, u_i) \in (C_i \times U_i)^{CBR}$. Let $i \in I$ be some player and take some $(c_i, u_i) \in (C_i \times U_i)^{CBR}$. Then, there exists an epistemic model $\mathcal{M}^\Gamma$ of Γ with a type $t_i \in T_i$ that expresses common belief in rationality such that c_i is optimal for (t_i, u_i). Take some $(c_j, t_j, u_j) \in C_j \times T_j \times U_j$ such that $b_i[t_i](c_j, t_j, u_j) > 0$. As t_i expresses common belief in rationality, t_j expresses common belief in rationality too, and c_j is optimal for (t_j, u_j). Thus, $(c_j, u_j) \in (C_j \times U_j)^{CBR}$, and, by the inductive assumption, $c_j \in C_j^k(u_j)$. Hence, for every choice $c_j \in \text{supp}(b_i[t_i])$ it is the case that $c_j \in C_j^k(u_j)$ for some utility function $u_j \in U_j$, and thus t_i only assigns positive probability to choices c_j contained in a decision problem $\Gamma_j^k(u_j)$ for some $u_j \in U_j$ for every opponent $j \in I \setminus \{i\}$. Consequently, t_i only assigns positive probability to choice combinations in $C_{-i}^{k+1}(u_i)$. Since c_i is optimal for

(t_i, u_i), it follows from Lemma 1 that $c_i \in C_i^{k+1}(u_i)$. Therefore, by induction, $(c_i, u_i) \in GISD_i$ obtains.

For the *if* direction of the theorem, suppose that the algorithm stops after $k \geq 0$ rounds. Then, for every $(c_i, u_i) \in GISD_i$ it is the case that $c_i \in C_i^k(u_i)$. By Lemma 1, c_i is optimal for some (p_i, u_i), where $p_i \in \Delta(C_{-i}^k(u_i))$. Observe that every $c_{-i} \in C_{-i}^k(u_i)$ only contains, for every player $j \in I \setminus \{i\}$, choices $c_j \in C_j$ such that $(c_j, u_j^{c_j}) \in GISD_j$ for some $u_j^{c_j} \in U_j$. Define a probability measure $p_i^{(c_i,u_i)} \in \Delta(GISD_{-i})$ by:

$$p_i^{(c_i,u_i)}(c_{-i}, u_{-i}) = \begin{cases} p_i(c_{-i}), & \text{if } c_{-i} \in C_{-i}^k(u_i) \text{ and } u_{-i} = u_{-i}^{c_{-i}} \\ 0, & \text{otherwise} \end{cases}$$

for all $(c_{-i}, u_{-i}) \in C_{-i} \times U_{-i}$. Construct an epistemic model $\mathcal{M}^\Gamma = \{(T_i)_{i \in I}, (b_i)_{i \in I}\}$ of Γ, where $T_i := \{t_i^{(c_i,u_i)} : (c_i, u_i) \in GISD_i\}$ for all $i \in I$ and:

$$b_i[t_i^{(c_i,u_i)}](c_{-i}, t_{-i}, u_{-i}) = \begin{cases} p_i^{(c_i,u_i)}(c_{-i}, u_{-i}), & \text{if } (c_{-i}, u_{-i}) \in GISD_{-i} \text{ and } t_j = t_j^{(c_j,u_j)} \text{ for all } j \in I \setminus \{i\} \\ 0, & \text{otherwise} \end{cases}$$

for all $(c_{-i}, t_{-i}, u_{-i}) \in C_{-i} \times T_{-i} \times U_{-i}$, for all $t_i^{(c_i,u_i)} \in T_i$ and for all $i \in I$. Observe that, by construction, for every player $i \in I$ and for every $(c_i, u_i) \in GISD_i$, the choice c_i is optimal for $(t_i^{(c_i,u_i)}, u_i)$. Hence, every type $t_i^{(c_i,u_i)}$ believes in the opponents' rationality. It then follows inductively that every such type $t_i^{(c_i,u_i)}$ also expresses common belief in rationality. Therefore, for every choice utility function pair $(c_i, u_i) \in GISD_i$, there exists a type $t_i^{(c_i,u_i)}$ within $\mathcal{M}^\Gamma$ such that $t_i^{(c_i,u_i)}$ expresses common belief in rationality and c_i is optimal for $(t_i^{(c_i,u_i)}, u_i)$. Hence, c_i is rational for u_i under common belief in rationality. ■

In terms of reasoning generalized iterated strict dominance thus corresponds to common belief in rationality. In fact, similar epistemic characterizations have been provided for the incomplete information solution concept of Δ-rationalizability in the literature. Notably, Battigalli and Siniscalchi (1999, Proposition 4), Battigalli (2003, Proposition 3.8) as well as Battigalli *et al.* (2011, p. 15) establish an equivalence between common belief in rationality and Δ-rationalizability.[3] It follows from these results in the literature and Theorem 1 that Δ-rationalizability and generalized iterated strict dominance are output equivalent, if no exogenous restrictions on the players' beliefs are imposed. The solution concept of interim correlated rationalizability due to Dekel *et al.* (2007) can also be epistemically characterized—for fixed marginal belief hierarchies on utilities—in terms of common belief in rationality (Battigalli *et al.*, 2011, Theorem 1). Due to the rigidity of marginal belief hierarchies on utilities, interim correlated rationalizability cannot be directly compared to Δ-rationalizability or generalized iterated strict dominance. However, if interim correlated rationalizability is applied to a given game for all possible marginal belief hierarchies on utilities, then the union of the corresponding solutions are equal to the output of Δ-rationalizability without any exogenous doxastic restrictions, and thus also to the output of generalized iterated strict dominance.

3 The special case of Δ-rationalizability which does not impose any exogenous restrictions on the players' beliefs is also called belief-free rationalizability and is explicitly characterized in terms of common belief in rationality by Battigalli *et al.* (2011, p. 14) too.

Besides the epistemic characterization in terms of iterated mutual belief in rationality, the solution concept of iterated strict dominance can also be characterized without recourse to any iterative procedure. An illuminating way of doing so is based on Pearce's (1984, Definition 2) complete information notion of best response property. Intuitively, a tuple of sets of choices of all players exhibits the best response property, whenever for every player the respective set only contains choices that are optimal for some belief about the opponents' choices only assigning positive probability to choices from the opponents' respective sets. In the context of incomplete information the idea of best response sets can then be formally defined as follows.

Definition 3 Let $\Gamma = (I, (C_i)_{i\in I}, (U_i)_{i\in I})$ be a static game with incomplete information, and $D_i \subseteq C_i \times U_i$ a set of choice utility function pairs for every player $i \in I$. A tuple $(D_i)_{i\in I}$ is called best response set tuple, if there exists, for every player $i \in I$ and for every choice utility function pair $(c_i, u_i) \in D_i$, a probability measure $\mu_i \in \Delta(D_{-i})$ such that c_i is optimal for (μ_i, u_i).

Similarly to Pearce's (1984, Proposition 2) characterization of his iterated procedure of rationalizability, our solution concept of generalized iterated strict dominance can also be shown to be equivalent to a best response set formulation.

Theorem 2 Let $\Gamma = (I, (C_i)_{i\in I}, (U_i)_{i\in I})$ be a static game with incomplete information, $i \in I$ some player, $c_i \in C_i$ some choice of player i, and $u_i \in U_i$ some utility function of player i. There exists a best response set tuple $(D_i)_{i\in I}$ such that $(c_i, u_i) \in D_i$, if and only if, $(c_i, u_i) \in GISD_i$.

Proof. For the *only if* direction of the theorem it is shown, by induction on $k \geq 0$, that $c_i \in C_i^k(u_i)$ for all $(c_i, u_i) \in D_i$, for all $k \geq 0$ and for all $i \in I$. Let $i \in I$ be some player and $(c_i, u_i) \in D_i$. It then holds that $c_i \in C_i^0(u_i) = C_i$. Now, consider some $(c_i, u_i) \in D_i$ and assume that $k \geq 0$ is such that $c_j \in C_j^k(u_j)$ for every $j \in I$ and for every $(c_j, u_j) \in D_j$. Fix some $(c_i, u_i) \in D_i$, and note that c_i is optimal for (μ_i, u_i), where $\mu_i \in \Delta(D_{-i})$ is some probability measure. By the inductive assumption, $c_j \in C_j^k(u_j)$ for every $(c_j, u_j) \in D_j$ and for every $j \in I \setminus \{i\}$. Hence, μ_i only assigns positive probability to opponents' choices $c_j \in C_j$ which are contained in $C_j^k(u_j)$ for some $u_j \in U_j$. Therefore, μ_i only assigns positive probability to opponents' choice combinations $c_{-i} \in C_{-i}^{k+1}(u_i)$. It follows, by Lemma 1, that c_i is not strictly dominated in the decision problem $(C_i^k(u_i), C_{-i}^{k+1}(u_i), u_i)$. Thus, $c_i \in C_i^{k+1}(u_i)$ and, by induction on $k \geq 0$, it holds that $(c_i, u_i) \in GISD_i$.

For the *if* direction of the theorem, it is shown that $(GISD_i)_{i\in I}$ is a best response set tuple. For every $u_j \in U_j$, let $C_j^*(u_j) := \{c_j \in C_j : (c_j, u_j) \in GISD_j\}$ and $C_j^* := \{c_j \in C_j : (c_j, u_j) \in GISD_j \text{ for some } u_j \in U_j\}$. Fix $(c_i, u_i) \in GISD_i$. Consequently, c_i is not strictly dominated in the decision problem $(C_i^*(u_i), C_{-i}^*, u_i)$. By Lemma 1, c_i is optimal for (p_i, u_i) for some $p_i \in \Delta(C_{-i}^*)$. Hence, c_i is optimal for (μ_i, u_i) for some $\mu_i \in \Delta(GISD_{-i})$. Therefore $(GISD_i)_{i\in I}$ is a best response set tuple. ■

Besides, it is actually the case that the algorithm of generalized iterated strict dominance always yields the largest best response set tuple as output.

Corollary 1 *Let* $\Gamma = (I, (C_i)_{i\in I}, (U_i)_{i\in I})$ be a static game with incomplete information. The set $GISD \subseteq \times_{i\in I}(C_i \times U_i)$ is the largest best response set tuple.

Proof. Let $i \in I$ be some player. By the proof of the *if*-direction of Theorem 2, $(GISD_j)_{j\in I}$ is a best response set tuple. Consider some element $(c_i, u_i) \in D_i$ of a best response set tuple $(D_j)_{j\in I}$ for player i. By Theorems 1 and 2, it follows that $(c_i, u_i) \in GISD_i$. Hence, $GISD_i$ is the largest best response set tuple for player i. ■

Accordingly, every best response set tuple is included in *GISD* and thus the set *GISD* can be interpreted as the largest fixed point of the generalized iterated strict dominance algorithm.

Since the solution concept of generalized iterated strict dominance corresponds to common belief in terms of reasoning by Theorem 1, it directly follows that rational choice for some utility function under common belief in rationality can also be given a non-iterative characterization in terms of the best response property.

Remark 2 Let $\Gamma = (I, (C_i)_{i\in I}, (U_i)_{i\in I})$ be a static game with incomplete information, $i \in I$ some player, $c_i \in C_i$ some choice of player i, and $u_i \in U_i$ some utility function of player i. There exists a best response set tuple $(D_i)_{i\in I}$ such that $(c_i, u_i) \in D_i$, if and only if, c_i is rational for u_i under common belief in rationality.

5. Complete information

The assumption of complete information eliminates any uncertainty about the payoff structure of the game. Formally, complete information constitutes the special case of incomplete information with the sets of utility functions all being singletons for every player. In line with a one-person perspective approach the question can be posed what conditions on the thinking of the reasoner in incomplete information games dissolve payoff uncertainty in his mind. Before tackling this issue, the notion of complete information needs to be formally defined within the framework of epistemic models.

Intuitively, complete information signifies that there exists no uncertainty about any opponent's utility function at any level of the reasoner's interactive thinking. Given some player $i \in I$, a type utility function pair $(t_i, u_i) \in T_i \times U_i$ can then be said to express complete information, if there exists for every opponent $j \in I \setminus \{i\}$ a utility function $u_j \in U_j$ such that t_i's marginal belief hierarchy t_i^U on utilities is generated by $(u_i, (u_j)_{j\in I\setminus\{i\}})$. That is, $b_i[t_i]((u_j)_{j\in I\setminus\{i\}}) = 1$, for every opponent $j \in I \setminus \{i\}$ player i only deems possible types $t_j \in T_j$ such that $b_j[t_j]((u_k)_{k\in I\setminus\{j\}}) = 1$, and for every opponent $j \in I \setminus \{i\}$ player i only deems possible types $t_j \in T_j$ that for every opponent $k \in I \setminus \{j\}$ only deem possible types $t_k \in T_k$ such that $b_k[t_k]((u_l)_{l\in I\setminus\{k\}}) = 1$, etc. Note that complete information is not defined simply for a type but for a type utility function tuple with the reasoner's actual utility function.

Also, the notion of correct beliefs needs to be invoked in the context of the players' utility functions. A type utility function tuple (t_i, u_i) is said to believe some opponent j to be correct about his utility function and marginal belief hierarchy t_i^U on utilities, if t_i only deems possible types t_j such that $b_j[t_j](u_i) = 1$ and $b_j[t_j]$ assigns probability 1 to t_i^U. Compared to complete information correct beliefs are defined for a type utility function tuple instead of merely for a type, since correct beliefs in the context of payoff uncertainty also concern the reasoner's utility function. With complete information and correct beliefs formally defined, the following theorem characterizes complete information by means of three doxastic correctness conditions.

Theorem 3 Let $\Gamma = (I, (C_i)_{i \in I}, (U_i)_{i \in I})$ be a static game with incomplete information, $\mathcal{M}^\Gamma$ some epistemic model of it, $i \in I$ some player, $t_i \in T_i$ some type of player i, and $u_i \in U_i$ some utility function of player i. The type utility function tuple (t_i, u_i) expresses complete information, if and only if,

- for every opponent $j \in I \setminus \{i\}$, type t_i only deems possible types $t_j \in T_j$ that are correct about i's utility function u_i and marginal belief hierarchy on utilities (Condition 1),
- for every opponent $j \in I \setminus \{i\}$, type t_i only deems possible type utility function pairs $(t_j, u_j) \in T_j \times U_j$ that only deem possible types $t_i' \in T_i$ that are correct about j's utilities and j's marginal belief hierarchy on utilities (Condition 2),
- for all opponents $j \in I \setminus \{i\}$ *and* $k \in \setminus \{i, j\}$, type t_i only deems possible types $t_j \in T_j$ that have the same marginal belief on k's utilities and on k's marginal belief hierarchies on utilities as t_i has (Condition 3).

Proof. Since only t_i's marginal belief hierarchy on utilities is affected by incomplete information and the three doxastic conditions, attention can be restricted to the induced marginal type t_i^U.

For the *if* direction of the theorem suppose that *i*'s utility function is $u_i \in U_i$ and that t_i satisfies the three conditions. It is first shown that t_i's marginal type t_i^U only deems possible a unique marginal type t_j^U and a unique utility function $u_j \in U_j$ for every opponent $j \in I \setminus \{i\}$. Towards a contradiction assume that t_i^U assigns positive probability to at least two marginal type utility function pairs (t_j^U, u_j) and $(t_j^{U'}, u_j')$ for some opponent $j \in I \setminus \{i\}$. Since t_i believes that *j* is correct about his utility function and marginal belief hierarchy on utilities, t_i believes that *j* only deems possible (t_i^U, u_i). Consequently, the marginal type utility function pairs (t_j^U, u_j) and $(t_j^{U'}, u_j')$ both only deem possible (t_i^U, u_i). Consider marginal type t_j^U and note that (t_j^U, u_j) believes that *i* deems it possible that *j* is characterized by the marginal type utility function tuple $(t_j^{U'}, u_j')$. Hence, (t_j^U, u_j) does not believe that *i* is correct about his utility function and marginal belief hierarchy on utilities. It follows that t_i deems it possible that *j* does not believe that *i* is correct about his utility function and marginal belief hierarchy on utilities, a contradiction. For every opponent $j \in I \setminus \{i\}$, type t_i's marginal type t_i^U thus assigns probability 1 to a single marginal type utility function tuple (t_j^U, u_j) and the corresponding type t_j assigns probability 1 to (t_i^U, u_i). By the third condition in Theorem 3 it is ensured that for each opponent the respective other opponents share the same marginal belief on utilities, thus it follows, by induction, that t_i's marginal belief hierarchy on utilities is generated by $(u_j)_{j \in I}$, and therefore (t_i, u_i) expresses complete information.

For the *only if* direction of the theorem, suppose that (t_i, u_i) expresses complete information and let $(u_j)_{j \in I} \in \times_{j \in I} U_j$ be the tuple of utility functions generating t_i's marginal belief hierarchy on utilities. Then, it directly follows by construction that the three doxastic correctness conditions hold. ■

From a conceptual point of view complete information can thus be modelled entirely within the mind of the reasoner satisfying the three conditions of Theorem 3 instead of restricting the game specification. Accordingly, the specific case of payoff certainty can also be obtained subjectively. In contrast, the objective realization of complete information restricts all players' sets of utility functions to singletons. Consequently, Theorem 3 can be interpreted as providing reasoning foundations for the complete information assumption in games from a one-person perspective.

From a technical point of view the question emerges whether the three doxastic correctness conditions in Theorem 3 are independent from each other. Since these conditions only affect the marginal belief hierarchies on types and utility functions, the independence issue can be investigated without reference to choices of any underlying game.

First of all, consider some three player game with $I = \{Alice, Bob, Claire\}$, $U_{Alice} = \{u_{Alice}\}$, $U_{Bob} = \{u_{Bob}\}$, and $U_{Claire} = \{u_{Claire}, u'_{Claire}\}$, as well as some epistemic model of the game with $T_{Alice} = \{t_{Alice}\}$, $T_{Bob} = \{t_{Bob}\}$, and $T_{Claire} = \{t_{Claire}\}$. The induced probability measures are defined as $b_{Alice}[t_{Alice}] = ((t_{Bob}, u_{Bob}), (t_{Claire}, u_{Claire}))$, $b_{Bob}[t_{Bob}] = ((t_{Alice}, u_{Alice}), (t_{Claire}, u'_{Claire}))$, and $b_{Claire}[t_{Claire}] = ((t_{Alice}, u_{Alice}), (t_{Bob}, u_{Bob}))$. Observe that the pair (t_{Alice}, u_{Alice}) satisfies Condition 1 and Condition 2 but violates Condition 3.

Secondly, consider some two player game with $I = \{Alice, Bob\}$, $U_{Alice} = \{u_{Alice}\}$, and $U_{Bob} = \{u_{Bob}, u'_{Bob}\}$, as well as some epistemic model of the game with $T_{Alice} = \{t_{Alice}\}$, and $T_{Bob} = \{t_{Bob}\}$. The induced probability measures are defined as $b_{Alice}[t_{Alice}] = \frac{1}{2}(t_{Bob}, u_{Bob}) + \frac{1}{2}(t_{Bob}, u'_{Bob})$, and $b_{Bob}[t_{Bob}] = (t_{Alice}, u_{Alice})$. Observes that the pair (t_{Alice}, u_{Alice}) satisfies Condition 1 and Condition 3 but violates Condition 2.

Thirdly, consider some two player game with $I = \{Alice, Bob\}$, $U_{Alice} = \{u_{Alice}, u'_{Alice}\}$, and $U_{Bob} = \{u_{Bob}\}$, as well as some epistemic model of the game with $T_{Alice} = \{t_{Alice}\}$, and $T_{Bob} = \{t_{Bob}\}$. The induced probability measures are defined as $b_{Alice}[t_{Alice}] = (t_{Bob}, u_{Bob})$, and $b_{Bob}[t_{Bob}] = \frac{1}{2}(t_{Alice}, u_{Alice}) + \frac{1}{2}(t_{Alice}, u'_{Alice})$. Observe that the pair (t_{Alice}, u_{Alice}) satisfies Condition 2 and Condition 3 but violates Condition 1.

It can thus be concluded that the three doxastic correctness conditions are independent from each other.

Generalized iterated strict dominance joins the class of solution concepts for static games with incomplete information. In fact, for complete information games the algorithm is equivalent to iterated strict dominance. To recall the definition of iterated strict dominance, let $\Gamma = (I, (C_i)_{i \in I}, (\{u_i\})_{i \in I})$ be a static game with complete information, and consider the sets $C_i^0 := C_i$ and

$$\begin{aligned} C_i^k := C_i^{k-1} \setminus \{c_i \in C_i : \text{ there exists } r_i \in \Delta(C_i^{k-1}) \\ \text{such that } u_i(c_i, c_{-i}) < \sum_{c'_i \in C_i} r_i(c'_i) \cdot u_i(c'_i, c_{-i}) \text{ for all } c_{-i} \in C_{-i}^{k-1}\} \end{aligned}$$

for all $k > 0$ and for all $i \in I$. The output of iterated strict dominance is then defined as $ISD := \times_{i \in I} ISD_i \subseteq \times_{i \in I} C_i$, where $ISD_i := \cap_{k \geq 0} C_i^k$ for every player $i \in I$. With complete information there is for every player i and for every round k a unique decision problem $\Gamma_i^k(u_i) = (C_i^k(u_i), C_{-i}^k(u_i), u_i)$, as payoff uncertainty vanishes. Thus, $C_{-i}^k(u_i) = \times_{j \in I \setminus \{i\}} C_j^k$, $C_i^k(u_i) = C_i^k$, and Definition 2 then becomes a formulation of iterated strict dominance in terms of decision problems. The following remark thus holds.

Remark 3 Let $\Gamma = (I, (C_i)_{i \in I}, (\{u_i\})_{i \in I})$ be a static game with complete information. Then, $\times_{i \in I} GISD_i = \times_{i \in I}(ISD_i \times \{u_i\})$.

Accordingly, generalized iterated strict dominance for incomplete information games with a single utility function for every player is equivalent to iterated strict dominance for complete information games. Therefore, our solution concept of generalized iterated strict dominance qualifies as the incomplete information analogue of iterated strict dominance for static games.

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