Exam EGT 2018

1 The Dancing Competition

Your desired grade is 8, Barbara's desired grade is 7.
Final grade is average of reported grades.
The closer the final grade to your desired grade, the better.

(a) Which choices are rational for you? For every rational choice, find belief that supports it. For every irrational choice, find another choice or randomized choice that str. dominates it.

A: Your rational choices:

10	optimal	for 6
9	"	" 7
8	"	" 8
7	"	" 9
6	"	" 10

Irrational choices: 1, -5, all str. dom. by 6

(b) Use an algorithm to find all choice that you and Barbara can rationally choose under which algorithm?

A:

Y			B		
10			~~10~~	②	str. dom. by 8
~~9~~	⑤	str. dom. by 10	~~9~~	②	
~~8~~	⑤		~~8~~	④	
~~7~~	③	str. dom. by 8	~~7~~	④	str. dom. by 6
~~6~~	③		~~6~~	⑥	
~~5~~	①		~~5~~	⑥	str. dom. by 4
~~4~~	①		4		
~~3~~	①	str. dom by 6	~~3~~	①	str. dom. by 4
~~2~~	①		~~2~~	①	
~~1~~	①		~~1~~	①	

For you: 10 For Barbara: 4.

New preferences: Your desired grade is 9

For you, $u_1 = 4 - 4.(\text{average grade} - \text{desired grade})^2$

For Barbara, $u_2 = 1$, if her reported grade is same as yours

$u_2 = 0$, otherwise.

Find choices that you and Barbara can rationally make under CBR.

A:

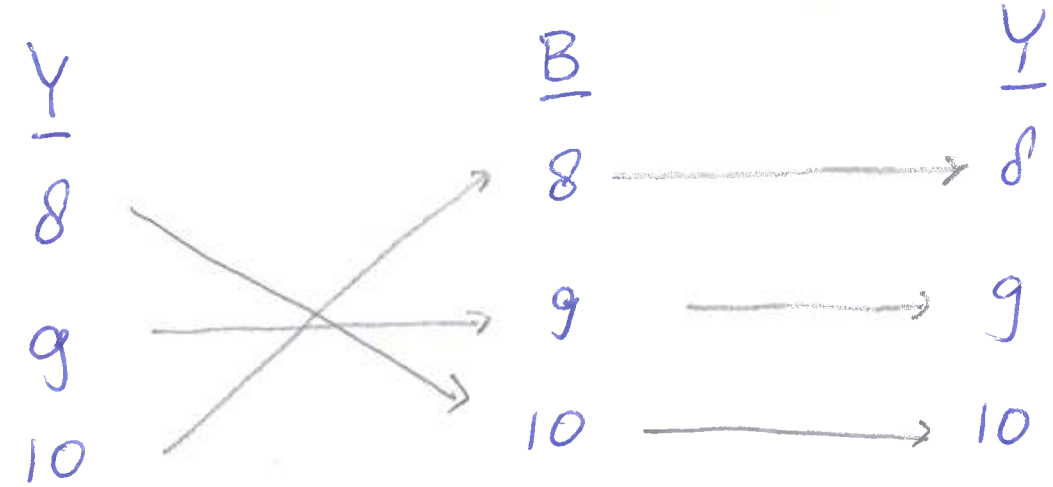
V		B	
10		10	
9		9	
8		8	
~~7~~	①	~~7~~	②
~~6~~	①	~~6~~	②
~~5~~	①	~~5~~	②
~~4~~	①	~~4~~	②
~~3~~	①	~~3~~	②
~~2~~	①	~~2~~	②
~~1~~	①	~~1~~	②

str. dom. by 8

str. dom. by $\frac{1}{3}10 + \frac{1}{3}9 + \frac{1}{3}8$

(d) Make a belief diagram (where every choice has ~~solid~~ outgoing arrow) found in Col ~~with only solid arrows~~. Which belief hierarchies for you express CBR? Which belief hierarchies for you are simple?

A:



All belief hierarchies express CBR. Only the one starting at 9 is simple.

(e) ~~Find choices you can rationally make under with sim~~

(e) Translate beliefs diagram into epistemic ma

A. Clear

(f) ~~Find choices you can rationally make~~ Show that under CBR with simple belief hierarchy, there is only one choice you can rationally make.

A:

	B: 8	9	10
8	0,1	3,0	4,0
A: 9	3,0	4,1	3,0
10	4,0	3,0	0,1

(9,9) is NE ⇒ you can rationally choose 9 under CBR with simple belief hierarch,

Show: (9,9) is only NE.

Suppose, $\sigma_1(8) > 0$ ⇒ 8 optimal for you under σ_2

⇒ $\boxed{\sigma_2(10) \geq \frac{1}{2}}$ ⇒ 10 optimal for B under σ_1

⇒ $\sigma_1(10) > 0$ ⇒ 10 optimal for you under σ_2

⇒ $\boxed{\sigma_2(8) \geq \frac{1}{2}}$

Therefore: $\tau_2(8) = \tau_2(10) = \frac{1}{2}$

$\Rightarrow u_1(8) = u_1(10) = 2 < u_1(9)$ ↯

Hence, $(9,9)$ is only NE.

(g) What extra condition is needed to get from CBR to NE? How appealing do you find this condition? Discuss.

2

(a)

You \ Barbara	0	10	20	30	40	
0	20,20	0,35	0,30	0,25	0,20	
10	35,0	15,15	0,25	0,20	0,15	
20	30,0	25,0	10,10	0,15	0,10	0,5
30	25,0	20,0	15,0	5,5	0,5	0,0
40	20,0	15,0	10,0	5,0	0,0	0,-5
50	15,0	10,0	5,0	0,0	-5,0	-5,-5

(b) ~~weakly~~ so strictly dom. by 40 for both.
Afterwards, nothing is str. dom.
Hence, you can rationally choose 0, 10, 20, 30, 40 under CBR

(c) -

(d) 0 weakly dom. by 10
10 optimal for (0; ...)
20 optimal for (10; ...)
30 optimal for (20; ...)
40 ~~optimal for (30;~~ weakly dom. by 30
50 weakly dom. by 40

Dekel-Fudenberg procedure:

Round 1: Eliminate 0, 40, 50

Round 2: 10 str. dom. by $\frac{1}{2} \cdot 20 + \frac{1}{2} \cdot 30$

→ eliminate 10

Round 3: 20 str. dom. by 30 → eliminate 20

Only 30 survives.

(f) Only 30

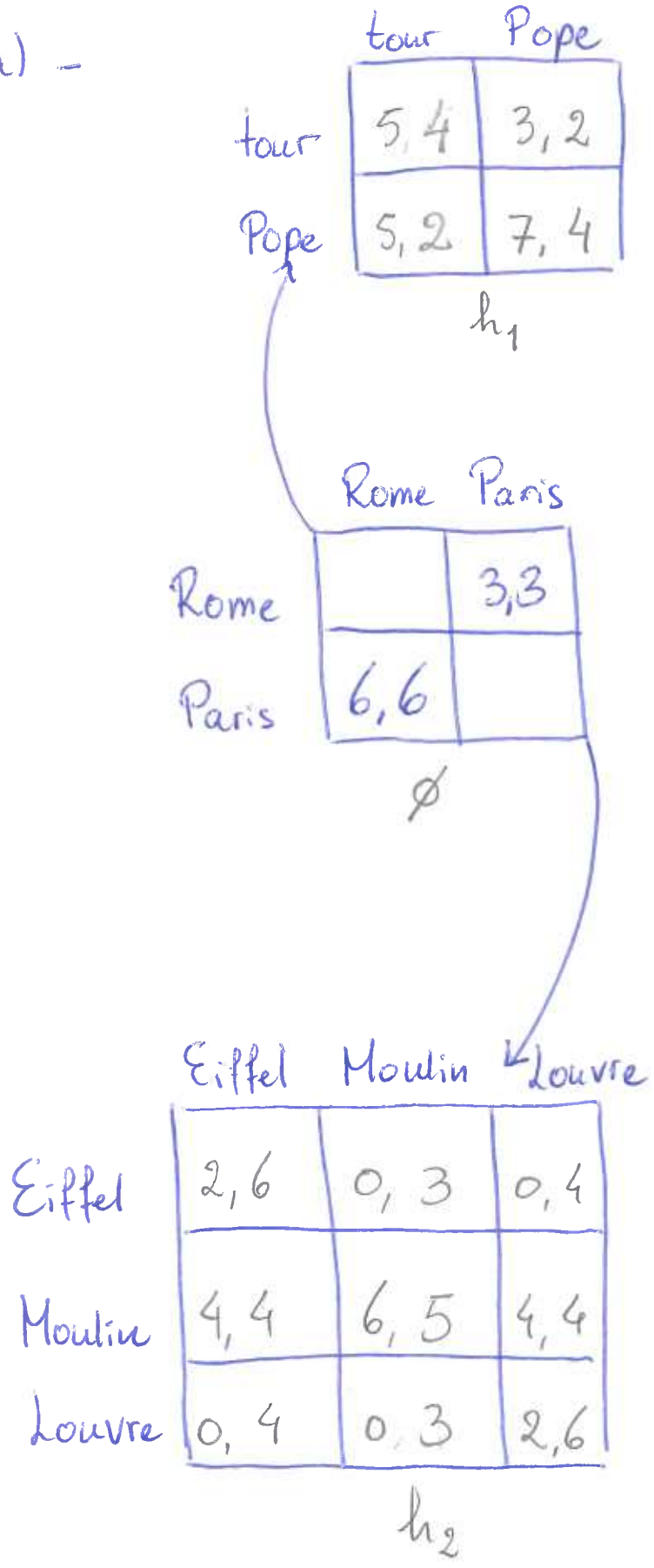
(g) $T_1 = \{t_1\}$. $T_2 = \{t_2\}$

$b_1(t_1) = ((30, t_2); (40, t_2); (20, t_2); (10; t_2); (0, t_2), (50, t_2))$

$b_2(t_2) = ((30, t_1); (40, t_1); (20, t_1); (10; t_1); (0, t_1); (50, t_1))$

3 (a) -

(b)


tour
Pope
tour
5,4
3,2
Pope
5,2
7,4
h_1
Rome
Paris
Rome
3,3
Paris
6,6
$\emptyset$
Eiffel
Moulin
Louvre
Eiffel
2,6
0,3
0,4
Moulin
4,4
6,5
4,4
Louvre
0,4
0,3
2,6
h_2

Backward dominance procedure with backwards order of elimination:

At h_1: Nothing can be eliminated

At h_2:

	(P,E) ②	(P,M)	(P,L) ②	
~~(P,E)~~	~~2,6~~	~~0,3~~	~~0,4~~	①
(P,M)	4,4	6,5	4,4	
~~(P,L)~~	~~0,4~~	~~0,3~~	~~2,6~~	①

Round 1: For you, (P,E) and (P,L) str. dom. by (P,M)

Round 2: For B, (P,E) and (P,L) str. dom. by (P,M)

At $\emptyset$:

	(R,T)	(R,P)	(P,M) ②	
~~(R,T)~~	~~5,4~~	~~3,2~~	~~3,3~~	①
(R,P)	5,2	7,4	3,3	
(P,M)	6,6	6,6	6,5	

Round 1: For you, (R,T) str. dom. by ~~$\frac{1}{2}(R,P)+\frac{1}{2}(P,M)$~~ (P,M)

~~For B, (P,M) str. dom. by $\frac{1}{2}(R,T)+\frac{1}{2}(R,P)$~~

Round 2: For B, (P,M) str. dom by (R,P)

Under CBFR, you can rationally choose (R, P) and (P, M), and Barbara can rationally choose (R, T) and (R, P).

(α)	∅	(R, T) ②	(R, P)	(P, E) ②	(P, M) ②	(P, L) ②
① ~~(R, T)~~	~~5, 4~~		~~3, 2~~	3, 3	3, 3	3, 3
(R, P)	5, 2		7, 4	3, 3	3, 3	3, 3
① ~~(P, E)~~	~~6, 6~~		~~6, 6~~	~~2, 6~~	~~0, 3~~	~~0, 4~~
③ ~~(P, M)~~	~~6, 6~~		~~6, 6~~	~~4, 4~~	~~6, 5~~	~~4, 4~~
① ~~(P, L)~~	~~6, 6~~		~~6, 6~~	0, 4	0, 3	2, 6

h_1	(R, T) ②	(R, P)
① ~~(R, T)~~	~~5, 4~~	~~3, 2~~
(R, P)	5, 2	7, 4

h_2	(P, E)	(P, M)	(P, L)
~~(P, E)~~	2, 6	~~0, 3~~	~~0, 4~~ ①
(P, M)	4, 4	6, 5	4, 4
~~(P, L)~~	0, 4	~~0, 3~~	~~2, 6~~ ①

Round 1: At ∅, for you, (R,T) str. dom. by (P,M)
→ eliminate (R,T) for you at ∅ and h_1.

~~At ∅, for B, (P,M) str. dom. by $\frac{1}{2}$(R,T)+$\frac{1}{2}$(R,P)~~
~~→ eliminate (P,M) for B at ∅ and h_2~~

At h_2, for you, (P,E) and (P,L) str. dom. by (P,M)
→ eliminate (P,E) and (P,L) for you at h_2 and ∅

Round 2: At ∅ ~~and h_2~~, for B, (P,E), and (P,L) str. dom. by (R,P) → eliminate (P,E), (P,M) and (P,L) for B at ∅ ~~and h_2~~ (but not at h_2)
~~At h_2, for B, (P,E) and (P,L) str. dom. by (P,M)~~
~~→ eliminate (P,E) and (P,L) for B at ∅ and h_2~~

At h_1, for B, (R,T) str. dom. by (R,P)
→ eliminate (R,T) for B at ∅ and h_1

Round 3: At ∅, for you, (P,M) str. dom by (R,P)
Eliminate (P,M) for you at ∅ (not at h_2).

Hence, under CSBR you and Barbara can only rationally choose (R, P).

(e) CBFR: Barbara believes at ∅ that you, in Paris, will go to Moulin Rouge. Hence, Barbara expects 5 at Paris.
Also, Barbara believes at ∅ that you, in Rome, will not choose tour (R,T), because it is worse than (P, M). Therefore, Barbara expects 4 in Rome.
As such, Rome is better for Barbara than Paris.
However, in Rome, Barbara could choose tour (because she may believe in Rome that you went to Rome by mistake, and will choose tour there).

Therefore, you can both choose Rome or Paris.
At Rome, you can both choose tour and pope.

CSBR: For you, (R, T) is worse than (P, M).
Therefore, if Barbara sees you in Rome, she will conclude that you choose pope, and she will choose pope as well. Since Barbara expects you to choose Moulin Rouge in Paris, Rome is better for B than Paris. Hence, you expect Barbara to choose Rome, and to choose pope in Rome. But then, you must choose Rome and pope.