EPICENTER Spring Course in Epistemic Game Theory

Maastricht University, July 1 – July 15, 2019
Exam

July 15, 9.00–13.00, Room A 1.22

Good luck!

Problem 1: The air balloon ride (35 points)

Your friend Deborah bought a ticket for an air balloon ride over the beautiful city of Maastricht. However, she recently discovered that she is afraid of heights, which of course is a huge problem for such a ride. Since you, Barbara and Chris would all love to see Maastricht from the air, Deborah decides to auction her air balloon ticket among the three of you.

The rules of the auction are as follows: You, Barbara and Chris each whisper a price in Deborah's ear. The price must be either $10, 20, 30, 40$ or 50 euros. The person with the highest price will get the ticket, and pays the price he or she whispered. If two persons whisper the same highest price, Deborah will toss a coin to decide who gets the ticket. If all three whisper the same price, Deborah will throw a dice, and each of you will get the ticket with probability $1/3$. Of course, you only pay the price in case you get the ticket.

Suppose that you and Barbara value the air balloon ticket at 31 euros, and that Chris values the ticket at 21 euros only. The utilities are as follows: If you win the ticket, your utility is your valuation of the ticket minus the price you pay. If you do not win the ticket, your utility is zero. Similarly for Barbara and Chris.

Please do not try to write down tables with utilities here, but try to do everything by reasoning instead.

(a) (5 points) Which prices are rational for you and which are not? For every rational price, find a belief about Barbara's and Chris' choice for which that price is optimal. For every irrational price, find another price, or randomization over prices, that strictly dominates it. Please explain your answers.

(b) (8 points) Which prices can you, Barbara and Chris rationally choose under common belief in rationality? What procedure do you use?
(Hint: In the third round of the procedure, certain prices are only strictly dominated by randomizations over prices.)

(c) (5 points) Create a beliefs diagram, with solid arrows only, in which you only use the prices found in (b).

(d) (4 points) Which of your belief hierarchies in this beliefs diagram express common belief in rationality? Which of these belief hierarchies is simple? (Hence, only focus on belief hierarchies for you). Explain your answers.

(e) (4 points) Translate your beliefs diagram from (c) into an epistemic model with types. Which of your types express common belief in rationality?

(f) (4 points) What prices can you, Barbara and Chris rationally choose under common belief in rationality with a simple belief hierarchy? What concept do you use here? Explain your answer.

(g) (5 points) Consider a general game with three players, i, j and k. Describe three important properties that a simple belief hierarchy for player i must have. Please discuss how reasonable you find each of these properties.

Problem 2: The angry farmer's brother (35 points)

Remember from the exercises "Stealing an apple" that you stole an apple from a farmer, after which the angry farmer is following you. You decided to hide in a big castle, from which you finally managed to escape. After leaving the castle the angry farmer is still chasing you, and you run into a new, smaller castle. There, things become even worse, because the owner of this smaller castle happens to be the angry farmer's brother, who is running after you with an ax. Below you find a map of that castle.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18

As you see, there are 18 chambers. The question for you is: In which chamber do you hide? And the question for the farmer's brother is: In which chamber will he search for you?

If the farmer's brother searches in a given chamber, he will find you whenever you hide in that particular chamber, or whenever you hide in a chamber that is horizontally, vertically or diagonally adjacent to it. For instance, if the farmer searches in chamber 9, he will find you whenever you hide in one of the chambers $2, 3, 4, 8, 9, 10, 14, 15$ or 16. Remember that this is the same as in the exercises "Stealing an apple" you have done in the course.

If the farmer's brother does not find you, your utility will be 1 and the farmer's brother's utility will be 0. If the farmer's brother finds you, your utility will be 0 and the farmer's brother's utility will be 1.

(a) (3 points) What choices can you and the farmer's brother rationally make under common belief in rationality with standard beliefs? Explain your answer.

We now turn to cautious reasoning with *lexicographic* beliefs.

(b) (6 points) What choices can you and the farmer's brother rationally make under common full belief in "caution and primary belief in rationality"? What procedure do you use?

(c) (8 points) What choices can you and the farmer's brother rationally make under common assumption of rationality? What procedure do you use?

Consider the following epistemic model with lexicographic beliefs, where there is only one type t_1 for you, and one type t_2 for the farmer's brother:

$$b_1(t_1) = ((11, t_2); (12, t_2); (18, t_2); (17, t_2); (2, t_2); (15, t_2); (8, t_2); ...)$$
$$b_2(t_2) = ((18, t_1); (4, t_1); (16, t_1); ...)$$

Here, the dots ... mean that the lexicographic beliefs of t_1 and t_2 are extended such that all opponent's choices are covered in the belief.

(d) (3 points) Which chamber is optimal for you if your type is t_1 ?

(e) (4 points) Is your type t_1 cautious? Does it primarily believe in the farmer's brother's rationality? Please explain.

(f) (6 points) Does your type t_1 respect the farmer's brother's preferences? Please explain.

(g) (5 points) Does your type t_1 assume the farmer's brother's rationality? Please explain.

Problem 3: Never let a lady wait again! (30 points)

Remember from the exercise "Never let a lady wait" that you and Barbara tried to have dinner together at the Italian or the Chinese restaurant. Things did not work out the way you planned, and at 9.00 pm you are still sitting in your favorite Italian restaurant, whereas Barbara is still waiting in her favorite Chinese restaurant. Half way between the Italian and Chinese restaurant there is a Greek restaurant, at half an hour walking distance from the Italian and Chinese restaurant. See the map below.

you   Barbara

●   ●   ●

Italian   Greek   Chinese

Barbara and you are both starving, and you want to have dinner together at one of the three restaurants, at 10.00 pm. Since you are both so hungry, it does not matter to you or Barbara at which restaurant you have dinner, as long as you are together.

The problem is that it is raining now, and it will only stop at 9.30 pm. Barbara has called you, and said she will only consider walking once it has stopped raining, at 9.30 pm. You, on the other hand, do consider walking in the rain, although you do not particularly like it. Hence, at 9.00 pm you have two options: You can either stay in the Italian restaurant, or walk to the Greek restaurant in the rain. You promised Barbara to call her at 9.30, to tell her where you are. After that phone call, at 9.30, you and Barbara will simultaneously decide whether to stay in the place where you are, or to walk to another restaurant within half an hour walking distance, without getting wet. Indeed, remember that it stops raining at 9.30 pm.

Since it is already late, the streets are very dark. Hence, if at 9.30 you decide to walk from the Greek restaurant to the Chinese restaurant, and Barbara walks from the Chinese restaurant to the Greek restaurant, you will not see each other.

If at 10.00 pm you and Barbara are at the same restaurant, you will both have your well deserved meal together, and your utility and Barbara's utility will increase by 4. However, walking for half an hour in the rain would decrease your utility by 3. On the other hand, walking for half an hour without getting wet only decreases your and Barbara's utility by 2.

(a) (3 points) Explain, in your own words, what the concept of common belief in future rationality means.

(b) (8 points) Model the story above as a dynamic game between you and Barbara. What are the possible strategies for you? And for Barbara? Is it a game with observed past choices? Is it a game with perfect information? Explain your answers.

(c) (6 points) What strategies can you and Barbara rationally choose under common belief in future rationality? What procedure do you use?

(d) (9 points) What strategies can you and Barbara rationally choose under common strong belief in rationality? What procedure do you use? What outcome do you expect?

(e) (4 points) Explain intuitively the reasoning behind common strong belief in rationality in this game, and how it leads to the outcome found in (d).

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