Exercises: Simple belief hierarchies

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a) The Game: Black or White?

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- You and Barbara are invited to a party.
- Both you and Barbara will have to independently decide whether to go to the party or not. And when you go, you both have to decide what to wear: black or white clothes.
- Conflicting interests about color of clothing: Barbara wants to match your color, you want to wear a different color from Barbara.

a) The Game: Black or White?

- You and Barbara are invited to a party.
- Both you and Barbara will have to independently decide whether to go to the party or not. And when you go, you both have to decide what to wear: black or white clothes.
- Conflicting interests about color of clothing: Barbara wants to match your color, you want to wear a different color from Barbara.
- Staying at home gives you a utility of 2. Going to the party and wearing a different color from Barbara gives you utility 3. In all other cases, you receive utility 0. Barbara has similar utilities. The only difference is that she gets a utility of 3 when she wears the same color as you do.

a) The Game: Black or White?

Barbara

You

a) The Game: Black or White?

Barbara

		Home	White	Black
You	Home	**2 , 2**	**2 , 0**	**2 , 0**
	White	**0 , 2**	**0 , 3**	**3 , 0**
	Black	**0 , 2**	**3 , 0**	**0 , 3**

b) Belief hierarchies: Extended beliefs diagram

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You	**Barbara**	**You**
Home	Home	Home
White	White	White
Black	Black	Black

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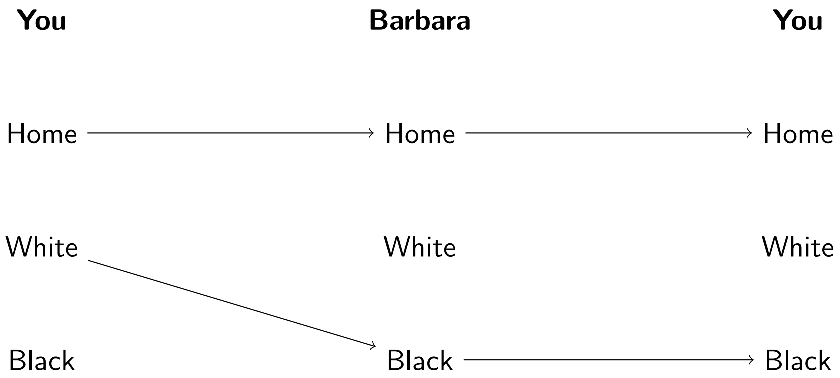

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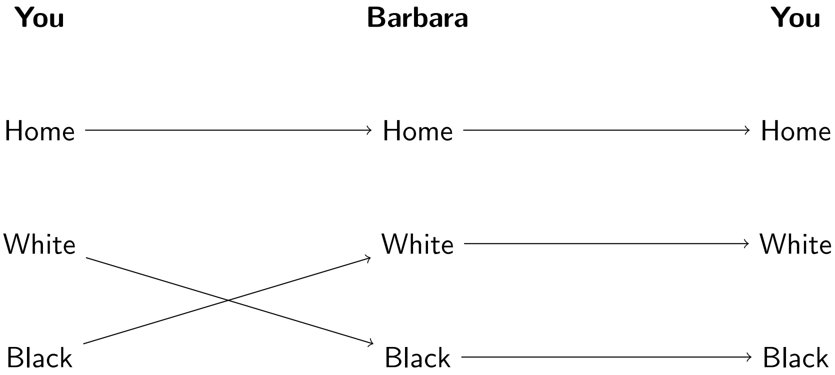

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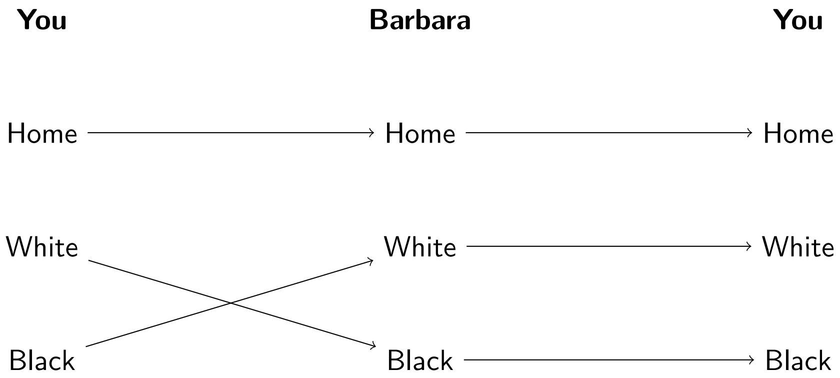

- All solid lines: All choices can be made under common belief in rationality!

b) Belief hierarchies: Epistemic model

Types	$T_y = \ldots$ $T_B = \ldots$		
	…	…	…
Beliefs for You	…	…	…
	…	…	…
	…	…	…
Beliefs for Barbara	…	…	…
	…	…	…

b) Belief hierarchies: Epistemic model

Types	$T_y = \{t_y^{Home}, t_y^{White}, t_y^{Black}\}$ $T_B = \{t_b^{Home}, t_b^{White}, t_b^{Black}\}$
Beliefs for You	$b_y(t_y^{Home}) = (Home, t_b^{Home})$ $b_y(t_y^{White}) = (Black, t_b^{Black})$ $b_y(t_y^{Black}) = (White, t_b^{White})$
Beliefs for Barbara	$b_b(t_b^{Home}) = (Home, t_y^{Home})$ $b_b(t_b^{White}) = (White, t_y^{White})$ $b_b(t_b^{Black}) = (Black, t_y^{Black})$

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Beliefs for You	$b_y(t_y^{Home})$	$=$	$(Home, t_b^{Home})$
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	$b_y(t_y^{Black})$	$=$	$(White, t_b^{White})$
Beliefs for Barbara	$b_b(t_b^{Home})$	$=$	$(Home, t_y^{Home})$
	$b_b(t_b^{White})$	$=$	$(White, t_y^{White})$
	$b_b(t_b^{Black})$	$=$	$(Black, t_y^{Black})$

- Types t_y^{Home}, t_y^{White}, t_y^{Black} express 1-fold belief in rationality.

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Beliefs for Barbara	$b_b(t_b^{Home}) = (Home, t_y^{Home})$ $b_b(t_b^{White}) = (White, t_y^{White})$ $b_b(t_b^{Black}) = (Black, t_y^{Black})$

- Types $t_y^{Home}, t_y^{White}, t_y^{Black}$ express 1-fold belief in rationality.
- Types $t_b^{Home}, t_b^{White}, t_b^{Black}$ express 1-fold belief in rationality.

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- Types $t_y^{Home}, t_y^{White}, t_y^{Black}$ express 1-fold belief in rationality.
- Types $t_b^{Home}, t_b^{White}, t_b^{Black}$ express 1-fold belief in rationality.
- All types in the model express 1-fold belief in rationality.
- Hence: every type expresses common belief in rationality!

c) Correct beliefs?

Types	$T_y = \{t_y^{Home}, t_y^{White}, t_y^{Black}\}$ $T_B = \{t_b^{Home}, t_b^{White}, t_b^{Black}\}$		
Beliefs for You	$b_y(t_y^{Home})$	$=$	$(Home, t_b^{Home})$
	$b_y(t_y^{White})$	$=$	$(Black, t_b^{Black})$
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Beliefs for Barbara	$b_b(t_b^{Home})$	$=$	$(Home, t_y^{Home})$
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- Types with correct beliefs?
  t_y^{Home}: You believe Barbara chooses Home and Barbara correctly believes you believe she chooses Home.

c) Correct beliefs?

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	$T_B = \{t_b^{Home}, t_b^{White}, t_b^{Black}\}$		
Beliefs for You	$b_y(t_y^{Home})$	$=$	$(Home, t_b^{Home})$
	$b_y(t_y^{White})$	$=$	$(Black, t_b^{Black})$
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Beliefs for Barbara	$b_b(t_b^{Home})$	$=$	$(Home, t_y^{Home})$
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- Types with correct beliefs?
  t_y^{Home}: You believe Barbara chooses Home and Barbara correctly believes you believe she chooses Home.

You **Barbara**

$$(Home, t_b^{Home}) \rightarrow (Home, t_y^{Home})$$

c) Correct beliefs?

- Type t_b^{Home} shows correct beliefs because of the same underlying reason.

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- Type t_b^{Home} shows correct beliefs because of the same underlying reason.
- Both t_y^{Home} and t_b^{Home} convey correct beliefs and only refer to each other → Belief that the opponent believes you have correct beliefs as well → simple belief hierarchy.

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- Type t_y^{White}:

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- Type t_y^{White}: You believe Barbara chooses *Black*, but also believe Barbara believes you believe she chooses *White*. X

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- Type t_y^{White}: You believe Barbara chooses *Black*, but also believe Barbara believes you believe she chooses *White*. X
- Type t_b^{White}: Barbara believes you choose *White*, but also believes you believe she believes you choose *Black*. X

c) Correct beliefs?

- Type t_b^{Home} shows correct beliefs because of the same underlying reason.
- Both t_y^{Home} and t_b^{Home} convey correct beliefs and only refer to each other → Belief that the opponent believes you have correct beliefs as well → simple belief hierarchy.

- Type t_y^{White}: You believe Barbara chooses *Black*, but also believe Barbara believes you believe she chooses *White*. X
- Type t_b^{White}: Barbara believes you choose *White*, but also believes you believe she believes you choose *Black*. X
- By symmetry one may confirm similar story for t_y^{Black} and t_b^{Black}.

d) Compute all Nash equilibria

		Barbara		
		Home	White	Black
You	Home	2 , 2	2 , 0	2 , 0
	White	0 , 2	0 , 3	3 , 0
	Black	0 , 2	3 , 0	0 , 3

- Pure NE:

d) Compute all Nash equilibria

		Barbara		
		Home	White	Black
	Home	2 , 2	2 , 0	2 , 0
You	White	0 , 2	0 , 3	3 , 0
	Black	0 , 2	3 , 0	0 , 3

- Pure NE:$(\sigma_y, \sigma_b) = (Home, Home)$

d) Compute all Nash equilibria

		Barbara		
		Home	White	Black
You	Home	2 , 2	2 , 0	2 , 0
	White	0 , 2	0 , 3	3 , 0
	Black	0 , 2	3 , 0	0 , 3

- Pure NE:$(\sigma_y, \sigma_b) = (Home, Home)$
- Corresponding simple belief hierarchy: $b_y(t_y^{Home}) = (Home, t_b^{Home})$ and $b_b(t_b^{Home}) = (Home, t_y^{Home})$.

d) Compute all Nash equilibria

		Barbara		
		Home	White	Black
You	Home	2 , 2	2 , 0	2 , 0
	White	0 , 2	0 , 3	3 , 0
	Black	0 , 2	3 , 0	0 , 3

- Pure NE:$(\sigma_y, \sigma_b) = (Home, Home)$
- Corresponding simple belief hierarchy: $b_y(t_y^{Home}) = (Home, t_b^{Home})$ and $b_b(t_b^{Home}) = (Home, t_y^{Home})$.
- What about mixed Nash equilibria?

d) Compute all Nash equilibria

Barbara

		Home	White	Black
You	Home	2 , 2	2 , 0	2 , 0
	White	0 , 2	0 , 3	3 , 0
	Black	0 , 2	3 , 0	0 , 3

d) Compute all Nash equilibria

Barbara

		Home	White	Black
You	Home	**2 , 2**	**2 , 0**	**2 , 0**
	White	**0 , 2**	**0 , 3**	**3 , 0**
	Black	**0 , 2**	**3 , 0**	**0 , 3**

- Suppose $\sigma_y(Black) > 0$

d) Compute all Nash equilibria

		Barbara		
		Home	White	Black
You	Home	**2 , 2**	**2 , 0**	**2 , 0**
	White	**0 , 2**	**0 , 3**	**3 , 0**
	Black	**0 , 2**	**3 , 0**	**0 , 3**

- Suppose $\sigma_y(Black) > 0 \rightarrow \sigma_b(White) \cdot 3 \geq 2$ (2 You are guaranteed from choosing Home).

d) Compute all Nash equilibria

Barbara

		Home	White	Black
	Home	**2 , 2**	**2 , 0**	**2 , 0**
You	White	**0 , 2**	**0 , 3**	**3 , 0**
	Black	**0 , 2**	**3 , 0**	**0 , 3**

- Suppose $\sigma_y(Black) > 0 \rightarrow \sigma_b(White) \cdot 3 \geq 2$ (2 You are guaranteed from choosing Home). $\rightarrow \sigma_b(White) \geq \frac{2}{3}$

d) Compute all Nash equilibria

Barbara

		Home	White	Black
You	Home	**2 , 2**	**2 , 0**	**2 , 0**
	White	**0 , 2**	**0 , 3**	**3 , 0**
	Black	**0 , 2**	**3 , 0**	**0 , 3**

- Suppose $\sigma_y(Black) > 0 \rightarrow \sigma_b(White) \cdot 3 \geq 2$ (2 You are guaranteed from choosing Home). $\rightarrow \sigma_b(White) \geq \frac{2}{3} \rightarrow \sigma_b(White) > 0$

d) Compute all Nash equilibria

Barbara

		Home	White	Black
You	Home	**2 , 2**	**2 , 0**	**2 , 0**
	White	**0 , 2**	**0 , 3**	**3 , 0**
	Black	**0 , 2**	**3 , 0**	**0 , 3**

- Suppose $\sigma_y(Black) > 0 \rightarrow \sigma_b(White) \cdot 3 \geq 2$ (2 You are guaranteed from choosing Home). $\rightarrow \sigma_b(White) \geq \frac{2}{3} \rightarrow \sigma_b(White) > 0 \rightarrow \sigma_y(White) \cdot 3 \geq 2$ (2 Barbara is guaranteed from choosing Home).

d) Compute all Nash equilibria

Barbara

		Home	White	Black
You	Home	**2 , 2**	**2 , 0**	**2 , 0**
	White	**0 , 2**	**0 , 3**	**3 , 0**
	Black	**0 , 2**	**3 , 0**	**0 , 3**

- Suppose $\sigma_y(Black) > 0 \rightarrow \sigma_b(White) \cdot 3 \geq 2$ (2 You are guaranteed from choosing Home). $\rightarrow \sigma_b(White) \geq \frac{2}{3} \rightarrow \sigma_b(White) > 0 \rightarrow \sigma_y(White) \cdot 3 \geq 2$ (2 Barbara is guaranteed from choosing Home). $\rightarrow \sigma_y(White) \geq \frac{2}{3}$

d) Compute all Nash equilibria

You		Barbara: Home	Barbara: White	Barbara: Black
	Home	**2 , 2**	**2 , 0**	**2 , 0**
	White	**0 , 2**	**0 , 3**	**3 , 0**
	Black	**0 , 2**	**3 , 0**	**0 , 3**

- Suppose $\sigma_y(Black) > 0 \rightarrow \sigma_b(White) \cdot 3 \geq 2$ (2 You are guaranteed from choosing Home). $\rightarrow \sigma_b(White) \geq \frac{2}{3} \rightarrow \sigma_b(White) > 0 \rightarrow \sigma_y(White) \cdot 3 \geq 2$ (2 Barbara is guaranteed from choosing Home). $\rightarrow \sigma_y(White) \geq \frac{2}{3} \rightarrow \sigma_b(Black) \geq \frac{2}{3}$

d) Compute all Nash equilibria

		Barbara		
		Home	White	Black
You	Home	**2 , 2**	**2 , 0**	**2 , 0**
	White	**0 , 2**	**0 , 3**	**3 , 0**
	Black	**0 , 2**	**3 , 0**	**0 , 3**

- Suppose $\sigma_y(Black) > 0 \rightarrow \sigma_b(White) \cdot 3 \geq 2$ (2 You are guaranteed from choosing Home). $\rightarrow \sigma_b(White) \geq \frac{2}{3} \rightarrow \sigma_b(White) > 0 \rightarrow \sigma_y(White) \cdot 3 \geq 2$ (2 Barbara is guaranteed from choosing Home). $\rightarrow \sigma_y(White) \geq \frac{2}{3} \rightarrow \sigma_b(Black) \geq \frac{2}{3} \rightarrow \sigma_b(Black) + \sigma_b(White) \geq \frac{2}{3} + \frac{2}{3} > 1$. Contradiction.

d) Compute all Nash equilibria

		Barbara: Home	Barbara: White	Barbara: Black
You	Home	**2 , 2**	**2 , 0**	**2 , 0**
	White	**0 , 2**	**0 , 3**	**3 , 0**
	Black	**0 , 2**	**3 , 0**	**0 , 3**

- Suppose $\sigma_y(Black) > 0 \rightarrow \sigma_b(White) \cdot 3 \geq 2$ (2 You are guaranteed from choosing Home). $\rightarrow \sigma_b(White) \geq \frac{2}{3} \rightarrow \sigma_b(White) > 0 \rightarrow \sigma_y(White) \cdot 3 \geq 2$ (2 Barbara is guaranteed from choosing Home). $\rightarrow \sigma_y(White) \geq \frac{2}{3} \rightarrow \sigma_b(Black) \geq \frac{2}{3} \rightarrow \sigma_b(Black) + \sigma_b(White) \geq \frac{2}{3} + \frac{2}{3} > 1$. Contradiction.
- So $\sigma_y(Black) = 0$.
- Because of symmetry in utilities $\sigma_y(White) = 0$

d) Compute all Nash equilibria

Barbara

		Home	White	Black
You	Home	**2 , 2**	**2 , 0**	**2 , 0**
	White	**0 , 2**	**0 , 3**	**3 , 0**
	Black	**0 , 2**	**3 , 0**	**0 , 3**

d) Compute all Nash equilibria

		Barbara		
		Home	White	Black
You	Home	**2 , 2**	**2 , 0**	**2 , 0**
	White	**0 , 2**	**0 , 3**	**3 , 0**
	Black	**0 , 2**	**3 , 0**	**0 , 3**

- As $\sigma_y(White) = \sigma_y(Black) = 0$, we have that $\sigma_y(Home) = 1$ must be the case.

d) Compute all Nash equilibria

		Barbara		
		Home	White	Black
You	Home	**2 , 2**	**2 , 0**	**2 , 0**
	White	**0 , 2**	**0 , 3**	**3 , 0**
	Black	**0 , 2**	**3 , 0**	**0 , 3**

- As $\sigma_y(White) = \sigma_y(Black) = 0$, we have that $\sigma_y(Home) = 1$ must be the case.
- $\sigma_y(Home) = 1 \rightarrow \sigma_b(Home) = 1$.

d) Compute all Nash equilibria

		Barbara Home	Barbara White	Barbara Black
You	Home	**2 , 2**	**2 , 0**	**2 , 0**
	White	**0 , 2**	**0 , 3**	**3 , 0**
	Black	**0 , 2**	**3 , 0**	**0 , 3**

- As $\sigma_y(White) = \sigma_y(Black) = 0$, we have that $\sigma_y(Home) = 1$ must be the case.
- $\sigma_y(Home) = 1 \rightarrow \sigma_b(Home) = 1$.
- If You choose to stay Home with certainty, Barbara cannot do better than staying Home as well.

d) Compute all Nash equilibria

- Pure NE $(\sigma_y, \sigma_b) = (Home, Home)$ is the only NE.
- Corresponding simple belief hierarchy:
  $b_y(t_y^{Home}) = (Home, t_b^{Home})$ and $b_b(t_b^{Home}) = (Home, t_y^{Home})$.

e) All rational choices under CBR with simple belief hierarchies

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The rational choices under a simple belief hierarchy that expresses common belief in rationality are exactly the Nash choices. So staying Home is the only rational choice for both you and Barbara.

Break time!

The Game

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- Number 1,2,3,...,30.

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The Game

- Tomorrow there is a reunion of your class, which had 30 students.
- Number 1,2,3,...,30.
- Every person i has a favorite classmate $i+1$ and a least favorite classmate $i-1$. If $i=1$, then the least favorite classmate is 30 and if $i=30$ then the favorite classmate is 1.
- Staying at home gives a utility of 2 to each person i; going to the reunion when the favorite classmate is present as well increases utility with 3; the presence of the least favorite classmate decreases utility with 3.

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- Optimal to stay home: E.g. you believe your favorite classmate stays home.→ That person believes that his favorite classmate stays home.→ *That* person believes that *his* favorite classmate stays home, and so on.

a) Both going and staying are optimal under CBR

- Optimal to stay home: E.g. you believe your favorite classmate stays home.→ That person believes that his favorite classmate stays home.→ *That* person believes that *his* favorite classmate stays home, and so on. Then at every level of your belief hierarchy you believe the person i in question believes his favorite classmate $i+1$ stays home, making staying at home optimal for i as well.

a) Both going and staying are optimal under CBR

- Optimal to stay home: E.g. you believe your favorite classmate stays home.→ That person believes that his favorite classmate stays home.→ *That* person believes that *his* favorite classmate stays home, and so on. Then at every level of your belief hierarchy you believe the person i in question believes his favorite classmate $i+1$ stays home, making staying at home optimal for i as well.
- Optimal to go: You must believe your favorite classmate goes and your least favorite classmate stays home.

a) Both going and staying are optimal under CBR

- Optimal to stay home: E.g. you believe your favorite classmate stays home.→ That person believes that his favorite classmate stays home.→ *That* person believes that *his* favorite classmate stays home, and so on. Then at every level of your belief hierarchy you believe the person i in question believes his favorite classmate $i+1$ stays home, making staying at home optimal for i as well.
- Optimal to go: You must believe your favorite classmate goes and your least favorite classmate stays home. → Your favorite person must assume that his favorite classmate goes and that you stay, etc.

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- Optimal to stay home: E.g. you believe your favorite classmate stays home.→ That person believes that his favorite classmate stays home.→ *That* person believes that *his* favorite classmate stays home, and so on. Then at every level of your belief hierarchy you believe the person i in question believes his favorite classmate $i+1$ stays home, making staying at home optimal for i as well.
- Optimal to go: You must believe your favorite classmate goes and your least favorite classmate stays home. → Your favorite person must assume that his favorite classmate goes and that you stay, etc.

What if one person does not have a favorite classmate?

a) Both going and staying are optimal under CBR

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- Optimal to go: You must believe your favorite classmate goes and your least favorite classmate stays home. → Your favorite person must assume that his favorite classmate goes and that you stay, etc.

What if one person does not have a favorite classmate? This is equivalent to saying that a person i's classmate $i+1$ is believed to certainly stay home by i.

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- Optimal to stay home: E.g. you believe your favorite classmate stays home.→ That person believes that his favorite classmate stays home.→ *That* person believes that *his* favorite classmate stays home, and so on. Then at every level of your belief hierarchy you believe the person i in question believes his favorite classmate $i+1$ stays home, making staying at home optimal for i as well.
- Optimal to go: You must believe your favorite classmate goes and your least favorite classmate stays home. → Your favorite person must assume that his favorite classmate goes and that you stay, etc.

What if one person does not have a favorite classmate? This is equivalent to saying that a person i's classmate $i+1$ is believed to certainly stay home by i. → Then staying home is optimal for i, causing a chain reaction.

b) Belief hierarchy expressing CBR

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For every classmate i, let us have:

Types:

$$T_i = \{t_i^g, t_i^s\}$$

Beliefs:

$$b_i(t_i^g) = (\ldots, (go, t_{i+1}^g), (stay, t_{i-1}^s), \ldots)$$
$$b_i(t_i^s) = (\ldots, (stay, t_{i+1}^s), \ldots)$$

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$$b_i(t_i^g) = (\ldots, (go, t_{i+1}^g), (stay, t_{i-1}^s), \ldots)$$
$$b_i(t_i^s) = (\ldots, (stay, t_{i+1}^s), \ldots)$$

For t_i^g, going is optimal as your favorite classmate goes and least favorite stays. For t_i^s, staying is optimal as your favorite classmate stays as well.

b) Belief hierarchy expressing CBR

For every classmate i, let us have:

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$$T_i = \{t_i^g, t_i^s\}$$

Beliefs:

$$b_i(t_i^g) = (\ldots, (go, t_{i+1}^g), (stay, t_{i-1}^s), \ldots)$$
$$b_i(t_i^s) = (\ldots, (stay, t_{i+1}^s), \ldots)$$

For t_i^g, going is optimal as your favorite classmate goes and least favorite stays. For t_i^s, staying is optimal as your favorite classmate stays as well.
Say you are player 1. If n classmates go to the party and $29 - n$ classmates do not, we have:

b) Belief hierarchy expressing CBR

For every classmate i, let us have:

Types:

$$T_i = \{t_i^g, t_i^s\}$$

Beliefs:

$$b_i(t_i^g) = (\ldots, (go, t_{i+1}^g), (stay, t_{i-1}^s), \ldots)$$
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For t_i^g, going is optimal as your favorite classmate goes and least favorite stays. For t_i^s, staying is optimal as your favorite classmate stays as well.
Say you are player 1. If n classmates go to the party and $29 - n$ classmates do not, we have:

$$b_1(t_1) = ((go, t_2^g), \ldots, (go, t_{n+1}^g), (stay, t_{n+2}^s), \ldots, (stay, t_{30}^s))$$

c) Rational choices under simple belief hierarchies expressing CBR

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- *Stay* is a rational choice under a simple belief hierarchy expressing CBR.
- Any other simple belief hierarchies expressing CBR?

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- $(\sigma_1, \sigma_2, \sigma_3, ..., \sigma_{30}) = (stay, stay, stay, ..., stay)$ induces a simple belief hierarchy that expresses CBR.
- *Stay* is a rational choice under a simple belief hierarchy expressing CBR.
- Any other simple belief hierarchies expressing CBR? Perhaps a hierarchy with probabilistic first-order beliefs?

c) Rational choices under simple belief hierarchies expressing CBR

- We will show that $(\sigma_1, \sigma_2, ..., \sigma_{30}) = (stay, stay, ..., stay)$ is the only NE. We do so by deriving a contradiction.

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- $\sigma_2(go) \geq \frac{2}{3}$ → You believe classmate 3 believes Prob(2 goes) $\geq \frac{2}{3}$ → You believe classmate 3 stays as a result → $\sigma_3(go) = 0$ → You believe classmate 2 stays → $\sigma_2(go) = 0$.
- Contradiction

D) How many classmates will show up?

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The only Nash choice for every person is to Stay home. So no person at all will show up.