Exercises: Respect of opponent's preferences

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July 8, 2019

6.3a) The game:

- You and Barbara are living in the same street and won a beautiful tree in a lottery.
- You have to decide where to plant it: Location A up to G.
- You live at location B, Barbara at location F.
- Distance between locations is 200 meters.
- If the tree is planted both you and Barbara have utility 6. If not, you both get utility 0.
- The tree is only planted if you decide on the same location, or neighbouring locations. In case of the latter, the tree is planted in the middle.
- For each 100 meters that the tree is planted from your and Barbara's house, you lose utility of 1.

6.3a) The game

6.3a) The game

		Barbara						
		a_2	b_2	c_2	d_2	e_2	f_2	g_2
You	a_1	**4,-4**	**5,-3**	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**
	b_1	**5,-3**	**6,-2**	**5,-1**	**0,0**	**0,0**	**0,0**	**0,0**
	c_1	**0,0**	**5,-1**	**4,0**	**3,1**	**0,0**	**0,0**	**0,0**
	d_1	**0,0**	**0,0**	**3,1**	**2,2**	**1,3**	**0,0**	**0,0**
	e_1	**0,0**	**0,0**	**0,0**	**1,3**	**0,4**	**-1,5**	**0,0**
	f_1	**0,0**	**0,0**	**0,0**	**0,0**	**-1,5**	**-2,6**	**-3,5**
	g_1	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**-3,5**	**-4,4**

6.3b) Rational choices under common full belief in C and ROP

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1	**4,-4**	**5,-3**	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**
b_1	**5,-3**	**6,-2**	**5,-1**	**0,0**	**0,0**	**0,0**	**0,0**
c_1	**0,0**	**5,-1**	**4,0**	**3,1**	**0,0**	**0,0**	**0,0**
d_1	**0,0**	**0,0**	**3,1**	**2,2**	**1,3**	**0,0**	**0,0**
e_1	**0,0**	**0,0**	**0,0**	**1,3**	**0,4**	**-1,5**	**0,0**
f_1	**0,0**	**0,0**	**0,0**	**0,0**	**-1,5**	**-2,6**	**-3,5**
g_1	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**-3,5**	**-4,4**

Which locations can you choose rationally under common full belief in caution and respect of the opponent's preferences.?

6.3b) Rational choices under common full belief in C and ROP

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1	**4,-4**	**5,-3**	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**
b_1	**5,-3**	**6,-2**	**5,-1**	**0,0**	**0,0**	**0,0**	**0,0**
c_1	**0,0**	**5,-1**	**4,0**	**3,1**	**0,0**	**0,0**	**0,0**
d_1	**0,0**	**0,0**	**3,1**	**2,2**	**1,3**	**0,0**	**0,0**
e_1	**0,0**	**0,0**	**0,0**	**1,3**	**0,4**	**-1,5**	**0,0**
f_1	**0,0**	**0,0**	**0,0**	**0,0**	**-1,5**	**-2,6**	**-3,5**
g_1	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**-3,5**	**-4,4**

Which locations can you choose rationally under common full belief in caution and respect of the opponent's preferences.?
We will use iterated addition of preference restrictions to find these.

6.3b) Rational choices under CFBCROP, Step 1

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1	**4,-4**	**5,-3**	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**
b_1	**5,-3**	**6,-2**	**5,-1**	**0,0**	**0,0**	**0,0**	**0,0**
c_1	**0,0**	**5,-1**	**4,0**	**3,1**	**0,0**	**0,0**	**0,0**
d_1	**0,0**	**0,0**	**3,1**	**2,2**	**1,3**	**0,0**	**0,0**
e_1	**0,0**	**0,0**	**0,0**	**1,3**	**0,4**	**-1,5**	**0,0**
f_1	**0,0**	**0,0**	**0,0**	**0,0**	**-1,5**	**-2,6**	**-3,5**
g_1	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**-3,5**	**-4,4**

- Preference restrictions for You:

6.3b) Rational choices under CFBCROP, Step 1

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1	**4,-4**	**5,-3**	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**
b_1	**5,-3**	**6,-2**	**5,-1**	**0,0**	**0,0**	**0,0**	**0,0**
c_1	**0,0**	**5,-1**	**4,0**	**3,1**	**0,0**	**0,0**	**0,0**
d_1	**0,0**	**0,0**	**3,1**	**2,2**	**1,3**	**0,0**	**0,0**
e_1	**0,0**	**0,0**	**0,0**	**1,3**	**0,4**	**-1,5**	**0,0**
f_1	**0,0**	**0,0**	**0,0**	**0,0**	**-1,5**	**-2,6**	**-3,5**
g_1	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**-3,5**	**-4,4**

- Preference restrictions for You:
  - $(a_1, \{b_1\})$: a_1 is weakly dominated by b_1.

6.3b) Rational choices under CFBCROP, Step 1

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1	**4,-4**	**5,-3**	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**
b_1	**5,-3**	**6,-2**	**5,-1**	**0,0**	**0,0**	**0,0**	**0,0**
c_1	**0,0**	**5,-1**	**4,0**	**3,1**	**0,0**	**0,0**	**0,0**
d_1	**0,0**	**0,0**	**3,1**	**2,2**	**1,3**	**0,0**	**0,0**
e_1	**0,0**	**0,0**	**0,0**	**1,3**	**0,4**	**-1,5**	**0,0**
f_1	**0,0**	**0,0**	**0,0**	**0,0**	**-1,5**	**-2,6**	**-3,5**
g_1	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**-3,5**	**-4,4**

- Preference restrictions for You:
  - $(a_1, \{b_1\})$: a_1 is weakly dominated by b_1.
  - $(g_1, \{e_1\})$: g_1 is weakly dominated by e_1.

6.3b) Rational choices under CFBCROP, Step 1

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1	**4,-4**	**5,-3**	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**
b_1	**5,-3**	**6,-2**	**5,-1**	**0,0**	**0,0**	**0,0**	**0,0**
c_1	**0,0**	**5,-1**	**4,0**	**3,1**	**0,0**	**0,0**	**0,0**
d_1	**0,0**	**0,0**	**3,1**	**2,2**	**1,3**	**0,0**	**0,0**
e_1	**0,0**	**0,0**	**0,0**	**1,3**	**0,4**	**-1,5**	**0,0**
f_1	**0,0**	**0,0**	**0,0**	**0,0**	**-1,5**	**-2,6**	**-3,5**
g_1	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**-3,5**	**-4,4**

- Preference restrictions for You:
  - $(a_1, \{b_1\})$: a_1 is weakly dominated by b_1.
  - $(g_1, \{e_1\})$: g_1 is weakly dominated by e_1.
  - $(f_1, \{e_1\})$, $(e_1, \{d_1\})$ $(e_1, \{c_1\})$, $(f_1, \{a_1\})$, $(g_1, \{a_1\})$.
  - (Transitivity: $(g_1, \{d_1\})$, $(g_1, \{c_1\})$, $(f_1, \{d_1\})$, $(f_1, \{c_1\})$, $(f_1, \{b_1\})$, $(g_1, \{b_1\})$).

6.3b) Rational choices under CFBCROP, Step 1

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1	**4,-4**	**5,-3**	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**
b_1	**5,-3**	**6,-2**	**5,-1**	**0,0**	**0,0**	**0,0**	**0,0**
c_1	**0,0**	**5,-1**	**4,0**	**3,1**	**0,0**	**0,0**	**0,0**
d_1	**0,0**	**0,0**	**3,1**	**2,2**	**1,3**	**0,0**	**0,0**
e_1	**0,0**	**0,0**	**0,0**	**1,3**	**0,4**	**-1,5**	**0,0**
f_1	**0,0**	**0,0**	**0,0**	**0,0**	**-1,5**	**-2,6**	**-3,5**
g_1	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**-3,5**	**-4,4**

- Preference restrictions for You:
  - $(a_1, \{b_1\})$: a_1 is weakly dominated by b_1.
  - $(g_1, \{e_1\})$: g_1 is weakly dominated by e_1.
  - $(f_1, \{e_1\})$, $(e_1, \{d_1\})$ $(e_1, \{c_1\})$, $(f_1, \{a_1\})$, $(g_1, \{a_1\})$.
  - (Transitivity: $(g_1, \{d_1\})$, $(g_1, \{c_1\})$, $(f_1, \{d_1\})$, $(f_1, \{c_1\})$, $(f_1, \{b_1\})$, $(g_1, \{b_1\})$).
- PR for Barbara by symmetry: $(g_2, \{f_2\})$, $(a_2, \{c_2\})$, $(b_2, \{c_2\})$, $(c_2, \{d_2\})$, $(c_2, \{e_2\})$, $(a_2, \{g_2\})$, $(b_2, \{g_2\})$.

6.3b) Rational choices under CFBCROP, Step 2

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1	**4,-4**	**5,-3**	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**
b_1	**5,-3**	**6,-2**	**5,-1**	**0,0**	**0,0**	**0,0**	**0,0**
c_1	**0,0**	**5,-1**	**4,0**	**3,1**	**0,0**	**0,0**	**0,0**
d_1	**0,0**	**0,0**	**3,1**	**2,2**	**1,3**	**0,0**	**0,0**
e_1	**0,0**	**0,0**	**0,0**	**1,3**	**0,4**	**-1,5**	**0,0**
f_1	**0,0**	**0,0**	**0,0**	**0,0**	**-1,5**	**-2,6**	**-3,5**
g_1	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**-3,5**	**-4,4**

- Preference restrictions for Barbara are $(g_2, \{f_2\})$, $(a_2, \{c_2\})$, $(b_2, \{c_2\})$, $(c_2, \{d_2\})$, $(c_2, \{e_2\})$, $(a_2, \{g_2\})$, $(b_2, \{g_2\})$.
- Take $(a_2, \{c_2\}), (b_2, \{c_2\}), (c_2, \{d_2\})$. Every likelihood ordering for you assumes set of choices with d_2, but not a_2, b_2, c_2.

6.3b) Rational choices under CFBCROP, Step 2

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1				**0,0**	**0,0**	**0,0**	**0,0**
b_1				**0,0**	**0,0**	**0,0**	**0,0**
c_1				**3,1**	**0,0**	**0,0**	**0,0**
d_1				**2,2**	**1,3**	**0,0**	**0,0**
e_1				**1,3**	**0,4**	**-1,5**	**0,0**
f_1				**0,0**	**-1,5**	**-2,6**	**-3,5**
g_1				**0,0**	**0,0**	**-3,5**	**-4,4**

- Preference restrictions for Barbara are $(g_2, \{f_2\})$, $(a_2, \{c_2\})$, $(b_2, \{c_2\})$, $(c_2, \{d_2\})$, $(c_2, \{e_2\})$, $(a_2, \{g_2\})$, $(b_2, \{g_2\})$.
- Take $(a_2, \{c_2\}), (b_2, \{c_2\}), (c_2, \{d_2\})$. Every likelihood ordering for you assumes set of choices with d_2, but not a_2, b_2, c_2.

6.3b) Rational choices under CFBCROP, Step 2

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1				**0,0**	**0,0**	**0,0**	**0,0**
b_1				**0,0**	**0,0**	**0,0**	**0,0**
c_1				**3,1**	**0,0**	**0,0**	**0,0**
d_1				**2,2**	**1,3**	**0,0**	**0,0**
e_1				**1,3**	**0,4**	**-1,5**	**0,0**
f_1				**0,0**	**-1,5**	**-2,6**	**-3,5**
g_1				**0,0**	**0,0**	**-3,5**	**-4,4**

- Preference restrictions for Barbara are $(g_2, \{f_2\})$, $(a_2, \{c_2\})$, $(b_2, \{c_2\})$, $(c_2, \{d_2\})$, $(c_2, \{e_2\})$, $(a_2, \{g_2\})$, $(b_2, \{g_2\})$.
- Take $(a_2, \{c_2\}), (b_2, \{c_2\}), (c_2, \{d_2\})$. Every likelihood ordering for you assumes set of choices with d_2, but not a_2, b_2, c_2.
- b_1 for You is weakly dominated by c_1 and d_1 and strictly worse given $d_2 \Rightarrow$ Add PRs $(b_1, \{c_1\}), (b_1, \{d_1\})$.

6.3b) Rational choices under CFBCROP, Step 2

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1	**4,-4**	**5,-3**	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**
b_1	**5,-3**	**6,-2**	**5,-1**	**0,0**	**0,0**	**0,0**	**0,0**
c_1	**0,0**	**5,-1**	**4,0**	**3,1**	**0,0**	**0,0**	**0,0**
d_1	**0,0**	**0,0**	**3,1**	**2,2**	**1,3**	**0,0**	**0,0**
e_1	**0,0**	**0,0**	**0,0**	**1,3**	**0,4**	**-1,5**	**0,0**
f_1	**0,0**	**0,0**	**0,0**	**0,0**	**-1,5**	**-2,6**	**-3,5**
g_1	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**-3,5**	**-4,4**

6.3b) Rational choices under CFBCROP, Step 2

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1	**4,-4**	**5,-3**	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**
b_1	**5,-3**	**6,-2**	**5,-1**	**0,0**	**0,0**	**0,0**	**0,0**
c_1	**0,0**	**5,-1**	**4,0**	**3,1**	**0,0**	**0,0**	**0,0**
d_1	**0,0**	**0,0**	**3,1**	**2,2**	**1,3**	**0,0**	**0,0**
e_1	**0,0**	**0,0**	**0,0**	**1,3**	**0,4**	**-1,5**	**0,0**
f_1	**0,0**	**0,0**	**0,0**	**0,0**	**-1,5**	**-2,6**	**-3,5**
g_1	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**-3,5**	**-4,4**

- By symmetry, for Barbara we add the Preference Restrictions $(f_2, \{e_2\}), (f_2, \{d_2\})$.

6.3b) Rational choices under CFBCROP, Step 3

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1	**4,-4**	**5,-3**	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**
b_1	**5,-3**	**6,-2**	**5,-1**	**0,0**	**0,0**	**0,0**	**0,0**
c_1	**0,0**	**5,-1**	**4,0**	**3,1**	**0,0**	**0,0**	**0,0**
d_1	**0,0**	**0,0**	**3,1**	**2,2**	**1,3**	**0,0**	**0,0**
e_1	**0,0**	**0,0**	**0,0**	**1,3**	**0,4**	**-1,5**	**0,0**
f_1	**0,0**	**0,0**	**0,0**	**0,0**	**-1,5**	**-2,6**	**-3,5**
g_1	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**-3,5**	**-4,4**

6.3b) Rational choices under CFBCROP, Step 3

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1	**4,-4**	**5,-3**	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**
b_1	**5,-3**	**6,-2**	**5,-1**	**0,0**	**0,0**	**0,0**	**0,0**
c_1	**0,0**	**5,-1**	**4,0**	**3,1**	**0,0**	**0,0**	**0,0**
d_1	**0,0**	**0,0**	**3,1**	**2,2**	**1,3**	**0,0**	**0,0**
e_1	**0,0**	**0,0**	**0,0**	**1,3**	**0,4**	**-1,5**	**0,0**
f_1	**0,0**	**0,0**	**0,0**	**0,0**	**-1,5**	**-2,6**	**-3,5**
g_1	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**-3,5**	**-4,4**

- Preference restrictions for Barbara: $(g_2, \{f_2\})$, $(a_2, \{c_2\})$, $(b_2, \{c_2\})$, $(c_2, \{d_2\})$, $(c_2, \{e_2\})$, $(a_2, \{g_2\})$, $(b_2, \{g_2\})$, $(f_2, \{e_2\})$, $(f_2, \{d_2\})$.
- Consider all of them together.

6.3b) Rational choices under CFBCROP, Step 3

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1				**0,0**	**0,0**		
b_1				**0,0**	**0,0**		
c_1				**3,1**	**0,0**		
d_1				**2,2**	**1,3**		
e_1				**1,3**	**0,4**		
f_1				**0,0**	**-1,5**		
g_1				**0,0**	**0,0**		

- Preference restrictions for Barbara: $(g_2, \{f_2\})$, $(a_2, \{c_2\})$, $(b_2, \{c_2\})$, $(c_2, \{d_2\})$, $(c_2, \{e_2\})$, $(a_2, \{g_2\})$, $(b_2, \{g_2\})$, $(f_2, \{e_2\})$, $(f_2, \{d_2\})$.
- Consider all of them together.

6.3b) Rational choices under CFBCROP, Step 3

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1				**0,0**	**0,0**		
b_1				**0,0**	**0,0**		
c_1				**3,1**	**0,0**		
d_1				**2,2**	**1,3**		
e_1				**1,3**	**0,4**		
f_1				**0,0**	**-1,5**		
g_1				**0,0**	**0,0**		

- Preference restrictions for Barbara: $(g_2, \{f_2\})$, $(a_2, \{c_2\})$, $(b_2, \{c_2\})$, $(c_2, \{d_2\})$, $(c_2, \{e_2\})$, $(a_2, \{g_2\})$, $(b_2, \{g_2\})$, $(f_2, \{e_2\})$, $(f_2, \{d_2\})$.
- Consider all of them together. Every remaining likelihood ordering must assume a set of choices that includes d_2 and e_2 but not a_2, b_2, c_2, f_2, g_2.
- Add $(b_1, \{e_1\})$ for You.

6.3b) Rational choices under CFBCROP, Step 3

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1	**4,-4**	**5,-3**	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**
b_1	**5,-3**	**6,-2**	**5,-1**	**0,0**	**0,0**	**0,0**	**0,0**
c_1	**0,0**	**5,-1**	**4,0**	**3,1**	**0,0**	**0,0**	**0,0**
d_1	**0,0**	**0,0**	**3,1**	**2,2**	**1,3**	**0,0**	**0,0**
e_1	**0,0**	**0,0**	**0,0**	**1,3**	**0,4**	**-1,5**	**0,0**
f_1	**0,0**	**0,0**	**0,0**	**0,0**	**-1,5**	**-2,6**	**-3,5**
g_1	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**-3,5**	**-4,4**

By symmetry, add for Barbara $(f_2, \{c_2\})$.

6.3b) Rational choices under CFBCROP, Stop

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1	**4,-4**	**5,-3**	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**
b_1	**5,-3**	**6,-2**	**5,-1**	**0,0**	**0,0**	**0,0**	**0,0**
c_1	**0,0**	**5,-1**	**4,0**	**3,1**	**0,0**	**0,0**	**0,0**
d_1	**0,0**	**0,0**	**3,1**	**2,2**	**1,3**	**0,0**	**0,0**
e_1	**0,0**	**0,0**	**0,0**	**1,3**	**0,4**	**-1,5**	**0,0**
f_1	**0,0**	**0,0**	**0,0**	**0,0**	**-1,5**	**-2,6**	**-3,5**
g_1	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**-3,5**	**-4,4**

6.3b) Rational choices under CFBCROP, Stop

	a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_1	**4,-4**	**5,-3**	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**
b_1	**5,-3**	**6,-2**	**5,-1**	**0,0**	**0,0**	**0,0**	**0,0**
c_1	**0,0**	**5,-1**	**4,0**	**3,1**	**0,0**	**0,0**	**0,0**
d_1	**0,0**	**0,0**	**3,1**	**2,2**	**1,3**	**0,0**	**0,0**
e_1	**0,0**	**0,0**	**0,0**	**1,3**	**0,4**	**-1,5**	**0,0**
f_1	**0,0**	**0,0**	**0,0**	**0,0**	**-1,5**	**-2,6**	**-3,5**
g_1	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**-3,5**	**-4,4**

- Preference restrictions for You: $(a_1, \{b_1\})$, $(g_1, \{e_1\})$, $(f_1, \{e_1\})$, $(e_1, \{d_1\})$ $(e_1, \{c_1\})$, $(f_1, \{a_1\})$, $(g_1, \{a_1\})$, $(b_1, \{c_1\})$, $(b_1, \{d_1\}$, $(b_1, \{e_1\})$.
- Preference restrictions for Barbara: $(g_2, \{f_2\})$, $(a_2, \{c_2\})$, $(b_2, \{c_2\})$, $(c_2, \{d_2\})$, $(c_2, \{e_2\})$, $(a_2, \{g_2\})$, $(b_2, \{g_2\})$, $(f_2, \{e_2\})$, $(f_2, \{d_2\})$, $(f_2, \{c_2\})$.
- Procedure stops here.

6.3b) Rational choices under CFBCROP, Stop

- You can rationally choose c_1 or d_1 under common full belief in caution and respect of opponent's preferences.
- Barbara can rationally choose d_2 or e_2 under common full belief in caution and respect of opponent's preferences.
- The remaining likelihood orderings for you are

6.3b) Rational choices under CFBCROP, Stop

- You can rationally choose c_1 or d_1 under common full belief in caution and respect of opponent's preferences.
- Barbara can rationally choose d_2 or e_2 under common full belief in caution and respect of opponent's preferences.
- The remaining likelihood orderings for you are

$$(\{d_2, e_2\}; c_2; f_2; g_2; a_2; b_2)$$
$$(d_2; e_2; c_2; f_2; g_2; a_2; b_2)$$
$$(e_2; d_2; c_2; f_2; g_2; a_2; b_2)$$

- The remaining likelihood orderings for Barbara are

6.3b) Rational choices under CFBCROP, Stop

- You can rationally choose c_1 or d_1 under common full belief in caution and respect of opponent's preferences.
- Barbara can rationally choose d_2 or e_2 under common full belief in caution and respect of opponent's preferences.
- The remaining likelihood orderings for you are

$$(\{d_2, e_2\}; c_2; f_2; g_2; a_2; b_2)$$
$$(d_2; e_2; c_2; f_2; g_2; a_2; b_2)$$
$$(e_2; d_2; c_2; f_2; g_2; a_2; b_2)$$

- The remaining likelihood orderings for Barbara are

$$(\{c_1, d_1\}; e_1; b_1; a_1; g_1; f_1)$$
$$(c_1; d_1; e_1; b_1; a_1; g_1; f_1)$$
$$(d_1; c_1; e_1; b_1; a_1; g_1; f_1)$$

6.3c) Epistemic model

We now need to build an epistemic model with types that optimize choices c_1 and d_1 for You and that express common belief in respect of opponent's preferences.
How?

6.3c) Epistemic model

We now need to build an epistemic model with types that optimize choices c_1 and d_1 for You and that express common belief in respect of opponent's preferences.
How? We can use the likelihood orderings for our lexicographic beliefs. Let us have a look at types t_Y^c and t_B^d, that optimize respectively choices c_1 for you and d_2 for Barbara.

6.3c) Epistemic model

We now need to build an epistemic model with types that optimize choices c_1 and d_1 for You and that express common belief in respect of opponent's preferences.
How? We can use the likelihood orderings for our lexicographic beliefs. Let us have a look at types t_Y^c and t_B^d, that optimize respectively choices c_1 for you and d_2 for Barbara.

Types	$T_Y = \{t_Y^c, t_Y^e$ $T_B = \{t_B^d, t_B^e\}$		
You	$b_Y(t_Y^c)$	$=$	$((d_2, t_B^d); (e_2, t_B^d); (c_2, t_B^d); (f_2, t_B^d); (g_2, t_B^d); (a_2, t_B^d); (b_2, t_B^d))$
	$b_Y(t_Y^d)$	$=$	$((e_2, t_B^e); (d_2, t_B^e); (c_2, t_B^e); (f_2, t_B^e); (g_2, t_B^e); (a_2, t_B^e); (b_2, t_B^e))$
Barbara	$b_B(t_B^d)$	$=$	$((c_1, t_Y^c); (d_1, t_Y^c); (e_1, t_Y^c); (b_1, t_Y^c); (a_1, t_Y^c); (g_1, t_Y^c); (f_1, t_Y^c)$
	$b_B(t_B^e)$	$=$	$((d_1, t_Y^d); (c_1, t_Y^d); (e_1, t_Y^d); (b_1, t_Y^d); (a_1, t_Y^d); (g_1, t_Y^d); (f_1, t_Y^d)$

6.3c) Epistemic model

	$T_Y = \{t_Y^c, t_Y^e$		
Types	$T_B = \{t_B^d, t_B^e\}$		
You	$b_Y(t_Y^c)$	$=$	$((d_2, t_B^d); (e_2, t_B^d); (c_2, t_B^d); (f_2, t_B^d); (g_2, t_B^d); (a_2, t_B^d); (b_2, t_B^d))$
	$b_Y(t_Y^d)$	$=$	$((e_2, t_B^e); (d_2, t_B^e); (c_2, t_B^e); (f_2, t_B^e); (g_2, t_B^e); (a_2, t_B^e); (b_2, t_B^e))$
Barbara	$b_B(t_B^d)$	$=$	$((c_1, t_Y^c); (d_1, t_Y^c); (e_1, t_Y^c); (b_1, t_Y^c); (a_1, t_Y^c); (g_1, t_Y^c); (f_1, t_Y^c)$
	$b_B(t_B^e)$	$=$	$((d_1, t_Y^d); (c_1, t_Y^d); (e_1, t_Y^d); (b_1, t_Y^d); (a_1, t_Y^d); (g_1, t_Y^d); (f_1, t_Y^d)$

Let us have a look at type t_Y^c, together with the game table: "Given d_2, we have that c_1 is best, then d_1 and then e_1. You are indifferent between all remaining choices. Given e_2 then, we see that f_1 should definitely come last in our ordering. Given c_2, we have that b_1 is the best choice. So b_1 should come after e_1. Given f_2, we have that f_2 should be next to last after f_1. That leaves us with choice a_1, which then has to go in between b_1 and g_1".

6.3c) Epistemic model

Types	$T_Y = \{t_Y^c, t_Y^e$ $T_B = \{t_B^d, t_B^e\}$		
You	$b_Y(t_Y^c)$	$=$	$((d_2, t_B^d); (e_2, t_B^d); (c_2, t_B^d); (f_2, t_B^d); (g_2, t_B^d); (a_2, t_B^d); (b_2, t_B^d))$
	$b_Y(t_Y^d)$	$=$	$((e_2, t_B^e); (d_2, t_B^e); (c_2, t_B^e); (f_2, t_B^e); (g_2, t_B^e); (a_2, t_B^e); (b_2, t_B^e))$
Barbara	$b_B(t_B^d)$	$=$	$((c_1, t_Y^c); (d_1, t_Y^c); (e_1, t_Y^c); (b_1, t_Y^c); (a_1, t_Y^c); (g_1, t_Y^c); (f_1, t_Y^c)$
	$b_B(t_B^e)$	$=$	$((d_1, t_Y^d); (c_1, t_Y^d); (e_1, t_Y^d); (b_1, t_Y^d); (a_1, t_Y^d); (g_1, t_Y^d); (f_1, t_Y^d)$

We exactly described here type t_B^d. So type t_B^d respects your preferences. We can conclude exactly the same for all other types. All types are cautious and respect opponent's preferences $\rightarrow$ CFBCROP.

Break time!

6.6a) The game:

- Chris will hold an auction for You and Barbara for his toy-car collection.
- You value the collection at 50 euros, Barbara at 40 euros.
- Bidding can only happen in multiples of 10, up to 60 euros. So $C_Y = C_B = \{0, 10, 20, 30, 40, 50, 60\}$.
- Your utility is $50 - P$ if you win the auction, 0 otherwise. Barbara's utility is $40 - P$ if she wins the auction, 0 otherwise.
- The winner is who bids most.
- In case of a tie, a fair coin toss will decide the winner (So 50-50).

6.6a) The game

	0	10	20	30	40	50	60
0	**25,20**	**0,30**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
10	**40,0**	**20,15**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**0,-5**	**0,-20**
60	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-5,-10**

6.6b) Rational choices under CFBCPR

Which prices can you rationally bid under common full belief in caution and primary belief in rationality? How would you find these?

6.6b) Rational choices under CFBCPR

Which prices can you rationally bid under common full belief in caution and primary belief in rationality? How would you find these?

We will use the Dekel-Fudenberg procedure:

6.6b) Rational choices under CFBCPR

Which prices can you rationally bid under common full belief in caution and primary belief in rationality? How would you find these?

We will use the Dekel-Fudenberg procedure:

- Step 1: Eliminate all weakly dominated choices.
- Step 2: In the reduced decision problem, eliminate all strictly dominated choices.
- ...

6.6b) Dekel-Fudenberg procedure, Step 1

	0	10	20	30	40	50	60
0	**25,20**	**0,30**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
10	**40,0**	**20,15**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**0,-5**	**0,-20**
60	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-5,-10**

Eliminate all weakly dominated choices.

- For You:

6.6b) Dekel-Fudenberg procedure, Step 1

	0	10	20	30	40	50	60
0	**25,20**	**0,30**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
10	**40,0**	**20,15**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**0,-5**	**0,-20**
60	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-5,-10**

Eliminate all weakly dominated choices.

- For You: 60 is weakly dominated by 50; 50 is weakly dominated by 40; 0 is weakly dominated by 10.
- For Barbara:

6.6b) Dekel-Fudenberg procedure, Step 1

	0	10	20	30	40	50	60
0	**25,20**	**0,30**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
10	**40,0**	**20,15**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**0,-5**	**0,-20**
60	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-5,-10**

Eliminate all weakly dominated choices.

- For You: 60 is weakly dominated by 50; 50 is weakly dominated by 40; 0 is weakly dominated by 10.
- For Barbara: 60 is weakly dominated by 50; 50 is weakly dominated by 40; 40 is weakly dominated by 30; 0 is weakly dominated by 10.

6.6b) Dekel-Fudenberg procedure, Step 2

	0	10	20	30	40	50	60
0							
10		**20,15**	**0,20**	**0,10**			
20		**30,0**	**15,10**	**0,10**			
30		**20,0**	**20,0**	**10,5**			
40		**10,0**	**10,0**	**10,0**			
50							
60							

Now eliminate all strictly dominated choices.

6.6b) Dekel-Fudenberg procedure, Step 2

	0	10	20	30	40	50	60
0							
10		**20,15**	**0,20**	**0,10**			
20		**30,0**	**15,10**	**0,10**			
30		**20,0**	**20,0**	**10,5**			
40		**10,0**	**10,0**	**10,0**			
50							
60							

Now eliminate all strictly dominated choices.

- For You:

6.6b) Dekel-Fudenberg procedure, Step 2

	0	10	20	30	40	50	60
0							
10		**20,15**	**0,20**	**0,10**			
20		**30,0**	**15,10**	**0,10**			
30		**20,0**	**20,0**	**10,5**			
40		**10,0**	**10,0**	**10,0**			
50							
60							

Now eliminate all strictly dominated choices.

- For You: 10 is strictly dominated by $0.5 \cdot \{30\} + 0.5 \cdot \{20\}$.
- For Barbara:

6.6b) Dekel-Fudenberg procedure, Step 2

	0	10	20	30	40	50	60
0							
10		**20,15**	**0,20**	**0,10**			
20		**30,0**	**15,10**	**0,10**			
30		**20,0**	**20,0**	**10,5**			
40		**10,0**	**10,0**	**10,0**			
50							
60							

Now eliminate all strictly dominated choices.

- For You: 10 is strictly dominated by $0.5 \cdot \{30\} + 0.5 \cdot \{20\}$.
- For Barbara: Nothing.

6.6b) Dekel-Fudenberg procedure, Step 3

	0	10	20	30	40	50	60
0							
10							
20		**30,0**	**15,10**	**0,10**			
30		**20,0**	**20,0**	**10,5**			
40		**10,0**	**10,0**	**10,0**			
50							
60							

Now eliminate again all strictly dominated choices.

6.6b) Dekel-Fudenberg procedure, Step 3

	0	10	20	30	40	50	60
0							
10							
20		**30,0**	**15,10**	**0,10**			
30		**20,0**	**20,0**	**10,5**			
40		**10,0**	**10,0**	**10,0**			
50							
60							

Now eliminate again all strictly dominated choices.

- For You:

6.6b) Dekel-Fudenberg procedure, Step 3

	0	10	20	30	40	50	60
0							
10							
20		**30,0**	**15,10**	**0,10**			
30		**20,0**	**20,0**	**10,5**			
40		**10,0**	**10,0**	**10,0**			
50							
60							

Now eliminate again all strictly dominated choices.

- For You: Nothing.
- For Barbara:

6.6b) Dekel-Fudenberg procedure, Step 3

	0	10	20	30	40	50	60
0							
10							
20		**30,0**	**15,10**	**0,10**			
30		**20,0**	**20,0**	**10,5**			
40		**10,0**	**10,0**	**10,0**			
50							
60							

Now eliminate again all strictly dominated choices.

- For You: Nothing.
- For Barbara: Nothing.

6.6b) Dekel-Fudenberg procedure, Conclusion

	0	10	20	30	40	50	60
0							
10							
20		**30,0**	**15,10**	**0,10**			
30		**20,0**	**20,0**	**10,5**			
40		**10,0**	**10,0**	**10,0**			
50							
60							

So for You are rational under CFBCPR: 20, 30 and 40. For Barbara we have: 10, 20, 30.

6.6c) Iterated addition of preference restrictions, Step 1

	0	10	20	30	40	50	60
0	**25,20**	**0,30**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
10	**40,0**	**20,15**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**0,-5**	**0,-20**
60	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-5,-10**

- Preference restrictions for You:

6.6c) Iterated addition of preference restrictions, Step 1

	0	10	20	30	40	50	60
0	**25,20**	**0,30**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
10	**40,0**	**20,15**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**0,-5**	**0,-20**
60	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-5,-10**

- Preference restrictions for You: $(60, \{50\})$, $(50, \{40\})$, $(50, \{30\})$ $(50, \{0\})$, $(0, \{10\})$, $(0, \{20\})$ (Red is "New").
- Preference restrictions for Barbara:

6.6c) Iterated addition of preference restrictions, Step 1

	0	10	20	30	40	50	60
0	**25,20**	**0,30**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
10	**40,0**	**20,15**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**0,-5**	**0,-20**
60	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-5,-10**

- Preference restrictions for You: $(60, \{50\})$, $(50, \{40\})$, $(50, \{30\})$ $(50, \{0\})$, $(0, \{10\})$, $(0, \{20\})$ (Red is "New").
- Preference restrictions for Barbara: $(60, \{50\})$, $(50, \{40\})$, $(40, \{30\})$, $(40, \{0\})$, $(0, \{10\})$, $(0, \{20\})$ (Red is "New")

6.6c) Iterated addition of preference restrictions, Step 2 (You)

	0	10	20	30	40	50	60
0	**25,20**	**0,30**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
10	**40,0**	**20,15**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**0,-5**	**0,-20**
60	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-5,-10**

- Preference restrictions for Barbara: $(60, \{50\})$, $(50, \{40\})$, $(40, \{30\})$, $(40, \{0\})$, $(0, \{10\})$, $(0, \{20\})$.
- Take $(60, \{50\})$, $(50, \{40\})$, $(40, \{0\})$, $(0, \{10\})$.

6.6c) Iterated addition of preference restrictions, Step 2 (You)

	0	10	20	30	40	50	60
0	**25,20**	**0,30**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
10	**40,0**	**20,15**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**0,-5**	**0,-20**
60	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-5,-10**

- Preference restrictions for Barbara: $(60, \{50\})$, $(50, \{40\})$, $(40, \{30\})$, $(40, \{0\})$, $(0, \{10\})$, $(0, \{20\})$.
- Take $(60, \{50\})$, $(50, \{40\})$, $(40, \{0\})$, $(0, \{10\})$.
- Every likelihood ordering for you that respects these preferences must assume a set of choices that includes 10 and 20, but not 0, 40, 50 or 60

6.6c) Iterated addition of preference restrictions, Step 2 (You)

	0	10	20	30	40	50	60
0		**0,30**	**0,20**	**0,10**			
10		**20,15**	**0,20**	**0,10**			
20		**30,0**	**15,10**	**0,10**			
30		**20,0**	**20,0**	**10,5**			
40		**10,0**	**10,0**	**10,0**			
50		**0,0**	**0,0**	**0,0**			
60		**-10,0**	**-10,0**	**-10,0**			

- New restrictions for You: $(10, \{20\})$, $(10, \{30\})$, $(0, \{30\})$, $(0, \{40\})$ and $(40, \{30\})$.
- E.g. 10 is weakly dominated 20 and given 10 for Barbara strictly worse. 10 is weakly dominated by 30 and given 20 for Barbara strictly worse.

6.6c) Iterated addition of preference restrictions, Step 2 (Barbara)

	0	10	20	30	40	50	60
0	**25,20**	**0,30**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
10	**40,0**	**20,15**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**0,-5**	**0,-20**
60	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-5,-10**

- Preference restrictions for You: $(60, \{50\})$, $(50, \{40\})$, $(50, \{30\})$ $(50, \{0\})$, $(0, \{10\})$, $(0, \{20\})$.
- Take $(0, \{10\})$, $(50, \{0\})$ and $(60, \{50\})$.
- Every likelihood ordering for Barbara that respects these preferences must assume a set of choices that includes 10 and not 0, 50 and 60.

6.6c) Iterated addition of preference restrictions, Step 2 (Barbara)

	0	10	20	30	40	50	60
0							
10	**40,0**	**20,15**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50							
60							

6.6c) Iterated addition of preference restrictions, Step 2 (Barbara)

	0	10	20	30	40	50	60
0							
10	**40,0**	**20,15**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50							
60							

- New preference restrictions for Barbara: $(10, \{20\})$ and $(0, \{30\})$.

6.6c) Iterated addition of preference restrictions, Step 3 (You)

	0	10	20	30	40	50	60
0	**25,20**	**0,30**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
10	**40,0**	**20,15**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**0,-5**	**0,-20**
60	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-5,-10**

- Barbara's PRs: $(60, \{50\})$, $(50, \{40\})$, $(40, \{30\})$, $(40, \{0\})$, $(0, \{10\})$, $(0, \{20\})$, $(10, \{20\})$, $(0, \{30\})$.
- Take $(60, \{50\})$, $(50, \{40\})$, $(40, \{0\})$, $(0, \{10\})$ and $(10, \{20\})$.
- Every likelihood ordering for You that respects these preferences must assume 20 and not $10, 0, 40, 50, 60$.

6.6c) Iterated addition of preference restrictions, Step 3 (You)

	0	10	20	30	40	50	60
0			**0,20**	**0,10**			
10			**0,20**	**0,10**			
20			**15,10**	**0,10**			
30			**20,0**	**10,5**			
40			**10,0**	**10,0**			
50			**0,0**	**0,0**			
60			**-10,0**	**-10,0**			

- New restrictions for You: $(20, \{30\})$ and $(10, \{40\})$.

6.6c) Iterated addition of preference restrictions, Step 3 (Barbara)

	0	10	20	30	40	50	60
0	**25,20**	**0,30**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
10	**40,0**	**20,15**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**0,-5**	**0,-20**
60	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-5,-10**

- Preference restrictions for You: $(60, \{50\})$, $(50, \{40\})$, $(50, \{30\})$, $(50, \{0\})$, $(0, \{10\})$, $(0, \{20\})$, $(10, \{20\})$, $(10, \{30\})$, $(0, \{30\})$, $(0, \{40\})$ and $(40, \{30\})$.
- Take $(60, \{50\})$, $(50, \{0\})$, $(0, \{10\})$, $(10, \{30\})$.
- Every likelihood ordering for Barbara that respect these preferences must assume 30 and not $10, 0, 50, 60$.

6.6c) Iterated addition of preference restrictions, Step 3 (Barbara)

	0	10	20	30	40	50	60
0							
10							
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50							
60							

- Add $(20, \{30\})$.

6.6c) Iterated addition of preference restrictions, Step 4 (You)

	0	10	20	30	40	50	60
0	**25,20**	**0,30**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
10	**40,0**	**20,15**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**0,-5**	**0,-20**
60	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-5,-10**

- Preference restrictions for Barbara: $(60, \{50\})$, $(50, \{40\})$, $(40, \{30\})$, $(40, \{0\})$, $(0, \{10\})$, $(0, \{20\})$, $(10, \{20\})$ $(40, \{10\})$ and $(20, \{30\})$.

6.6c) Iterated addition of preference restrictions, Step 4 (You)

	0	10	20	30	40	50	60
0	**25,20**	**0,30**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
10	**40,0**	**20,15**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**0,-5**	**0,-20**
60	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-5,-10**

- Preference restrictions for Barbara: $(60, \{50\})$, $(50, \{40\})$, $(40, \{30\})$, $(40, \{0\})$, $(0, \{10\})$, $(0, \{20\})$, $(10, \{20\})$ $(40, \{10\})$ and $(20, \{30\})$.
- These restrictions imply a unique likelihood ordering for You: $(30; 20; 10; 0; 40; 50; 60)$.

6.6c) Iterated addition of preference restrictions, Step 3 (You)

	0	10	20	30	40	50	60
0				**0,10**			
10				**0,10**			
20				**0,10**			
30				**10,5**			
40				**10,0**			
50				**0,0**			
60				**-10,0**			

6.6c) Iterated addition of preference restrictions, Step 3 (You)

	0	10	20	30	40	50	60
0				**0,10**			
10				**0,10**			
20				**0,10**			
30				**10,5**			
40				**10,0**			
50				**0,0**			
60				**-10,0**			

- New restriction for You: $(20, \{40\})$.

6.6c) Iterated addition of preference restrictions, Step 3 (You)

	0	10	20	30	40	50	60
0				**0,10**			
10				**0,10**			
20				**0,10**			
30				**10,5**			
40				**10,0**			
50				**0,0**			
60				**-10,0**			

- New restriction for You: $(20, \{40\})$.
- With this restriction we have a unique likelihood ordering for Barbara $(30; 40; 20; 10; 0; 50; 60)$.

6.6c) Iterated addition of preference restrictions, Stop

	0	10	20	30	40	50	60
0	**25,20**	**0,30**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
10	**40,0**	**20,15**	**0,20**	**0,10**	**0,0**	**0,-10**	**0,-20**
20	**30,0**	**30,0**	**15,10**	**0,10**	**0,0**	**0,-10**	**0,-20**
30	**20,0**	**20,0**	**20,0**	**10,5**	**0,0**	**0,-10**	**0,-20**
40	**10,0**	**10,0**	**10,0**	**10,0**	**5,0**	**0,-10**	**0,-20**
50	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**	**0,-5**	**0,-20**
60	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-10,0**	**-5,-10**

- We have the unique likelihood ordering for You: $(30; 20; 10; 0; 40; 50; 60)$.
- We have the unique likelihood ordering for Barbara: $(30; 40; 20; 10; 0; 50; 60)$.
- Then both Barbara and You can both only rationally choose under common full belief in caution and respect of opponent's preferences 30.

6.6d) Epistemic model

We now need to build an epistemic model with types that optimize bids 30 for You and Barbara and that express common belief in caution and ROP.
How?

6.6d) Epistemic model

We now need to build an epistemic model with types that optimize bids 30 for You and Barbara and that express common belief in caution and ROP.
How? We can do this by specifying a type for each choice identified in c), and define the type such that the induced belief corresponds to the likelihood orderings we found in c).

6.6d) Epistemic model

We now need to build an epistemic model with types that optimize bids 30 for You and Barbara and that express common belief in caution and ROP.

How? We can do this by specifying a type for each choice identified in c), and define the type such that the induced belief corresponds to the likelihood orderings we found in c).

	$T_Y = \{t_Y^{30}\}$		
Types	$T_B = \{t_B^{30}\}$		
You	$b_Y(t_Y^{30})$	$=$	$((30, t_B^{30}); (20, t_B^{30}); (10, t_B^{30}); (0, t_B^{30}); (40, t_B^{30}); (50, t_B^{30}); (60, t_B^{30}))$
Barbara	$b_B(t_B^{30})$	$=$	$((30, t_Y^{30}); (40, t_Y^{30}); (20, t_Y^{30}); (10, t_Y^{30}); (0, t_Y^{30}); (50, t_Y^{30}); (60, t_Y^{30}))$

6.6d) Epistemic model

	$T_Y = \{t_Y^{30}\}$		
Types	$T_B = \{t_B^{30}\}$		
You	$b_Y(t_Y^{30})$	$=$	$((30, t_B^{30}); (20, t_B^{30}); (10, t_B^{30}); (0, t_B^{30}); (40, t_B^{30}); (50, t_B^{30}); (60, t_B^{30}))$
Barbara	$b_B(t_B^{30})$	$=$	$((30, t_Y^{30}); (40, t_Y^{30}); (20, t_Y^{30}); (10, t_Y^{30}); (0, t_Y^{30}); (50, t_Y^{30}); (60, t_Y^{30}))$

Let us have a look at type t_Y^{30}, together with the game table: "Given that Barbara bids 30, both bids 30 and 40 for You give utility 10. And bid 60 is worst. Given that Barbara bids 20, bid 30 for You is better than bid 40. And next best is 20. Given that Barbara bids 10, we have that bid 10 should be preferred for you after 20. And given that Barbara bids 0, we have that bid 0 is better for You than bid 50."

6.6d) Epistemic model

	$T_Y = \{t_Y^{30}\}$		
Types	$T_B = \{t_B^{30}\}$		
You	$b_Y(t_Y^{30})$	$=$	$((30, t_B^{30}); (20, t_B^{30}); (10, t_B^{30}); (0, t_B^{30}); (40, t_B^{30}); (50, t_B^{30}); (60, t_B^{30}))$
Barbara	$b_B(t_B^{30})$	$=$	$((30, t_Y^{30}); (40, t_Y^{30}); (20, t_Y^{30}); (10, t_Y^{30}); (0, t_Y^{30}); (50, t_Y^{30}); (60, t_Y^{30}))$

This is exactly what type t_B^{30} specifies as a belief. Hence Barbara respects your preferences. We can do the same for type t_B^{30}, and conclude that You also respect Barbara's preferences. Each type respects each other's preferences, hence we have common belief in the opponent's preferences.