

Exercises: Common belief in rationality

Niels Mourmans

July 2, 2019

2.2a) The game:

- You are a piano student. In two weeks you have an exam.
- You know that two out of the three following musical pieces will be asked of you to play for a jury: An easy piece by Mozart (M), a more difficult piece of Chopin (C) and a very difficult piece by Rachmaninov (R).
- You will have to distribute your practicing accross two weeks. You can either choose to practice the same piece for two weeks or to practice two different pieces each for one week.
- The expected grade for Mozart is $4 + 3\sqrt{x}$; the expected grade for Chopin is $4 + 2.5x$; the expected grade for Rachmaninov is $4 + 1.5x^2$.
- Your utility the average of the two grades for the two pieces you have to play.
- The jury's utility is equal to the sum of your grade and 3 (C), 2 (R) or 1 (M).

2.2a) The game

2.2a) The game

		Jury		
		$M + C$	$M + R$	$C + R$
You	MM	6.12 , 10.12	6.12 , 9.12	4 , 9
	MC	6.75 , 10.75	5.5 , 8.5	5.25 , 10.25
	MR	5.5 , 9.5	6.25 , 9.25	4.75 , 9.75
	CC	6.5 , 10.5	4 , 7	6.5 , 11.5
	CR	5.25 , 9.25	4.75 , 7.75	6 , 11
	RR	4 , 8	7 , 10	7 , 12

2.2b) Rational choices for you

Which practice schedules for you can be rational?

2.2b) Rational choices for you

Which practice schedules for you can be rational?

- MM

2.2b) Rational choices for you

Which practice schedules for you can be rational?

- MM: $b_Y = 0.3(M + C) + 0.7(M + R)$. Practicing Mozart for two weeks is rational if you believe with probability 0.3 the jury will ask you to play Mozart and Chopin and with probability 0.7 Mozart and Rachmaninov;
- MC

2.2b) Rational choices for you

Which practice schedules for you can be rational?

- MM: $b_Y = 0.3(M + C) + 0.7(M + R)$. Practicing Mozart for two weeks is rational if you believe with probability 0.3 the jury will ask you to play Mozart and Chopin and with probability 0.7 Mozart and Rachmaninov;
- MC: $b_Y = (M + C)$;
- MR

2.2b) Rational choices for you

Which practice schedules for you can be rational?

- MM: $b_Y = 0.3(M + C) + 0.7(M + R)$. Practicing Mozart for two weeks is rational if you believe with probability 0.3 the jury will ask you to play Mozart and Chopin and with probability 0.7 Mozart and Rachmaninov;
- MC: $b_Y = (M + C)$;
- MR: **Nope**. Not a rational choice. We can think of a mixed choice between MM and RR that strictly dominates MR:

2.2b) Rational choices for you

Which practice schedules for you can be rational?

- MM: $b_Y = 0.3(M + C) + 0.7(M + R)$. Practicing Mozart for two weeks is rational if you believe with probability 0.3 the jury will ask you to play Mozart and Chopin and with probability 0.7 Mozart and Rachmaninov;
- MC: $b_Y = (M + C)$;
- MR: **Nope**. Not a rational choice. We can think of a mixed choice between MM and RR that strictly dominates MR: $0.74MM + 0.26RR$;
- CC

2.2b) Rational choices for you

Which practice schedules for you can be rational?

- MM: $b_Y = 0.3(M + C) + 0.7(M + R)$. Practicing Mozart for two weeks is rational if you believe with probability 0.3 the jury will ask you to play Mozart and Chopin and with probability 0.7 Mozart and Rachmaninov;
- MC: $b_Y = (M + C)$;
- MR: **Nope**. Not a rational choice. We can think of a mixed choice between MM and RR that strictly dominates MR: $0.74MM + 0.26RR$;
- CC: $b_Y = 0.5(M + C) + 0.5(C + R)$;
- CR

2.2b) Rational choices for you

Which practice schedules for you can be rational?

- MM: $b_Y = 0.3(M + C) + 0.7(M + R)$. Practicing Mozart for two weeks is rational if you believe with probability 0.3 the jury will ask you to play Mozart and Chopin and with probability 0.7 Mozart and Rachmaninov;
- MC: $b_Y = (M + C)$;
- MR: **Nope**. Not a rational choice. We can think of a mixed choice between MM and RR that strictly dominates MR: $0.74MM + 0.26RR$;
- CC: $b_Y = 0.5(M + C) + 0.5(C + R)$;
- CR: **Nope**. It is strictly dominated by $0.6CC + 0.4RR$.
- RR

2.2b) Rational choices for you

Which practice schedules for you can be rational?

- MM: $b_Y = 0.3(M + C) + 0.7(M + R)$. Practicing Mozart for two weeks is rational if you believe with probability 0.3 the jury will ask you to play Mozart and Chopin and with probability 0.7 Mozart and Rachmaninov;
- MC: $b_Y = (M + C)$;
- MR: **Nope**. Not a rational choice. We can think of a mixed choice between MM and RR that strictly dominates MR: $0.74MM + 0.26RR$;
- CC: $b_Y = 0.5(M + C) + 0.5(C + R)$;
- CR: **Nope**. It is strictly dominated by $0.6CC + 0.4RR$.
- RR: $b_Y = (C + R)$.

2.2c) Rational choices for the Jury

Which exams are rational for the jury?

2.2c) Rational choices for the Jury

Which exams are rational for the jury?

- M+C

2.2c) Rational choices for the Jury

Which exams are rational for the jury?

- M+C: $b_J = MC$;
- M+R

2.2c) Rational choices for the Jury

Which exams are rational for the jury?

- M+C: $b_J = MC$;
- M+R: **Nope**. Not a rational choice. We can think of a mixed choice that puts a little weight on M+C and the remaining weight on C+R, such that it strictly dominates M+R

2.2c) Rational choices for the Jury

Which exams are rational for the jury?

- M+C: $b_J = MC$;
- M+R: **Nope**. Not a rational choice. We can think of a mixed choice that puts a little weight on M+C and the remaining weight on C+R, such that it strictly dominates M+R:
 $0.2(M + C) + 0.8(C + R)$;
- C+R:

2.2c) Rational choices for the Jury

Which exams are rational for the jury?

- M+C: $b_J = MC$;
- M+R: **Nope**. Not a rational choice. We can think of a mixed choice that puts a little weight on M+C and the remaining weight on C+R, such that it strictly dominates M+R:
 $0.2(M + C) + 0.8(C + R)$;
- C+R: $b_J = RR$.

2.2d) Beliefs diagram

2.2d) Beliefs diagram

You	Barbara	You
MM	M+C	MM
MC		MC
MR		MR
CC	M+R	CC
CR		CR
RR	C+R	RR

3.2a) Extended beliefs diagram and epistemic model

3.2a) Extended beliefs diagram and epistemic model

You	Barbara	You
MM	M+C	MM
MC		MC
MR		MR
CC	M+R	CC
CR		CR
RR	C+R	RR

3.2a) Extended beliefs diagram and epistemic model

3.2a) Extended beliefs diagram and epistemic model

Types	$T_Y = \{t_Y^{MM}, t_Y^{MC}, t_Y^{MR}, t_Y^{CC}, t_Y^{CR}, t_Y^{RR}\}$	$T_J = \{t_J^{M+C}, t_J^{M+R}, t_J^{C+R}\}$
Beliefs for You	$b_Y(t_Y^{MM}) = 0.3[M + C, t_J^{M+C}] + 0.7[M + R, t_J^{M+R}]$	
	$b_Y(t_Y^{MC}) = [M + C, t_J^{M+C}]$	
X	$b_Y(t_Y^{MR}) = [M + R, t_J^{M+R}]$	
	$b_Y(t_Y^{CC}) = 0.5[M + C, t_J^{M+C}] + 0.5[C + R, t_J^{C+R}]$	
X	$b_Y(t_Y^{CR}) = [C + R, t_J^{C+R}]$	
	$b_Y(t_Y^{RR}) = [C + R, t_J^{C+R}]$	
Beliefs for Jury	$b_J(t_J^{M+C}) = [MC, t_Y^{MC}]$	
X	$b_J(t_J^{M+R}) = [MC, t_Y^{MC}]$	
	$b_J(t_J^{C+R}) = [RR, t_Y^{RR}]$	

We make use here of the beliefs identified in 2.2b and 2.2c

3.2b) Third-order beliefs for t_Y^{MM} and t_Y^{CC}

3.2b) Third-order beliefs for t_Y^{MM} and t_Y^{CC} For t_Y^{MM}

- First-level: You believe with probability 0.3 that the Jury will choose $M + C$ and believe with probability 0.7 that the Jury will choose $M + R$;

3.2b) Third-order beliefs for t_Y^{MM} and t_Y^{CC} For t_Y^{MM}

- First-level: You believe with probability 0.3 that the Jury will choose $M + C$ and believe with probability 0.7 that the Jury will choose $M + R$;
- Second-level: You believe with probability 0.3 that the Jury believes you will practice MC . And you believe with probability 0.7 that the Jury believes you will practice MC again. So in total you believe that the Jury believes you will practice MC ;

3.2b) Third-order beliefs for t_Y^{MM} and t_Y^{CC} For t_Y^{MM}

- First-level: You believe with probability 0.3 that the Jury will choose $M + C$ and believe with probability 0.7 that the Jury will choose $M + R$;
- Second-level: You believe with probability 0.3 that the Jury believes you will practice MC . And you believe with probability 0.7 that the Jury believes you will practice MC again. So in total you believe that the Jury believes you will practice MC ;
- Third-level: You believe with probability 0.3 that the Jury believes that you believe that the Jury will choose $M + C$. And you believe with probability 0.7 that the Jury believes that you believe that the Jury will choose $M + C$ again.

3.2b) Third-order beliefs for t_Y^{MM} and t_Y^{CC}

3.2b) Third-order beliefs for t_Y^{MM} and t_Y^{CC} For t_Y^{CC}

- First-level: You believe with probability 0.5 that the Jury will choose $M + C$ and believe with probability 0.5 the Jury will choose $C + R$;

3.2b) Third-order beliefs for t_Y^{MM} and t_Y^{CC} For t_Y^{CC}

- First-level: You believe with probability 0.5 that the Jury will choose $M + C$ and believe with probability 0.5 the Jury will choose $C + R$;
- Second-level: You believe with probability 0.5 that the Jury believes that you will practice MC . You believe with probability 0.5 that the Jury believes that you will practice RR ;

3.2b) Third-order beliefs for t_Y^{MM} and t_Y^{CC} For t_Y^{CC}

- First-level: You believe with probability 0.5 that the Jury will choose $M + C$ and believe with probability 0.5 the Jury will choose $C + R$;
- Second-level: You believe with probability 0.5 that the Jury believes that you will practice MC . You believe with probability 0.5 that the Jury believes that you will practice RR ;
- Third-level: You believe with probability 0.5 that the Jury believes that you believe that the Jury will choose $M + C$. You believe with probability 0.5 that the Jury believes that you believe that the Jury will choose $C + R$.

3.2c) 1-fold belief and 2-fold belief in rationality

3.2c) 1-fold belief and 2-fold belief in rationality

- Belief in the jury's rationality

3.2c) 1-fold belief and 2-fold belief in rationality

- Belief in the jury's rationality: Any type but t_Y^{MM} and t_Y^{MR} .
These two types believe the jury makes a choice that is never optimal (M+R).
- Belief that the jury believes in *your* rationality

3.2c) 1-fold belief and 2-fold belief in rationality

- Belief in the jury's rationality: Any type but t_Y^{MM} and t_Y^{MR} .
These two types believe the jury makes a choice that is never optimal (M+R).
- Belief that the jury believes in *your* rationality: Any type for the Jury. No type for the Jury refers to a choice that is never optimal. And we constructed each type such that it makes the choice we combine it with rational for that belief.
- Types that express common belief in rationality

3.2c) 1-fold belief and 2-fold belief in rationality

- Belief in the jury's rationality: Any type but t_Y^{MM} and t_Y^{MR} .
These two types believe the jury makes a choice that is never optimal (M+R).
- Belief that the jury believes in *your* rationality: Any type for the Jury. No type for the Jury refers to a choice that is never optimal. And we constructed each type such that it makes the choice we combine it with rational for that belief.
- Types that express common belief in rationality: t_Y^{MC} , t_Y^{CC} , t_Y^{CR} and t_Y^{RR} . Let us check this!

3.2d) Graphical method

3.2d) Graphical method

We will refer back to our extended beliefs diagram and adjust it a bit.

3.2d) Graphical method

We will refer back to our extended beliefs diagram and adjust it a bit.

We already established that MR and CR for you, as well as $M + R$ for the jury, are not rational choices. Then we just have to see if we can adjust the arrows from MM in such a way that MM is optimal for a belief that does not assign positive probability to $(M + R)$.

3.2d) Graphical method

We will refer back to our extended beliefs diagram and adjust it a bit.

We already established that MR and CR for you, as well as $M + R$ for the jury, are not rational choices. Then we just have to see if we can adjust the arrows from MM in such a way that MM is optimal for a belief that does not assign positive probability to $(M + R)$. This is never possible however. From all other remaining choices for You, following the lines we only come across solid lines.

3.2e) Elimination procedure

3.2e) Elimination procedure

We will now apply the **iterated elimination of strictly dominated choices** procedure. The choices that survive this procedure are rational choices under *some* belief hierarchy expressing common belief in rationality.

3.2e) Elimination procedure

We will now apply the **iterated elimination of strictly dominated choices** procedure. The choices that survive this procedure are rational choices under *some* belief hierarchy expressing common belief in rationality.

Recall the table we made in 2.2a

3.2e) Elimination procedure Step 1

		Jury		
		$M + C$	$M + R$	$C + R$
You	MM	6.12 , 10.12	6.12 , 9.12	4 , 9
	MC	6.75 , 10.75	5.5 , 8.5	5.25 , 10.25
	MR	5.5 , 9.5	6.25 , 9.25	4.75 , 9.75
	CC	6.5 , 10.5	4 , 7	6.5 , 11.5
	CR	5.25 , 9.25	4.75 , 7.75	6 , 11
	RR	4 , 8	7 , 10	7 , 12

What choices can we eliminate for You and Jury?

3.2e) Elimination procedure Step 1

		Jury		
		$M + C$	$M + R$	$C + R$
You	MM	6.12 , 10.12	6.12 , 9.12	4 , 9
	MC	6.75 , 10.75	5.5 , 8.5	5.25 , 10.25
	CC	6.5 , 10.5	4 , 7	6.5 , 11.5
	RR	4 , 8	7 , 10	7 , 12

We deleted MR and CR for You, as these were strictly domination (and thus irrational). What about the Jury's choices?

3.2e) Elimination procedure Step 1

		Jury		
		$M + C$	$M + R$	$C + R$
You	MM	6.12 , 10.12		4 , 9
	MC	6.75 , 10.75		5.25 , 10.25
	CC	6.5 , 10.5		6.5 , 11.5
	RR	4 , 8		7 , 12

We deleted $(M+R)$, as this was an irrational choices for the jury. This leads to a **reduced decision problem** Γ^1 , which we see here. Can we delete something further for You or the Jury?

3.2e) Elimination procedure Step 2

		Jury		
		$M + C$	$M + R$	$C + R$
You	MC	6.75 , 10.75		5.25 , 10.25
	CC	6.5 , 10.5		6.5 , 11.5
	RR	4 , 8		7 , 12

Yes, we see that MM is strictly dominated by MC . So we delete MM for You.

3.2e) Elimination procedure Step 2

		Jury		
		$M + C$	$M + R$	$C + R$
You	MC	6.75 , 10.75		5.25 , 10.25
	CC	6.5 , 10.5		6.5 , 11.5
	RR	4 , 8		7 , 12

Yes, we see that MM is strictly dominated by MC . So we delete MM for You. For the Jury nothing is strictly dominated. So we get a **new reduced decision problem** Γ^2 , which we see here.

3.2e) Elimination procedure Step 2

		Jury		
		$M + C$	$M + R$	$C + R$
You	MC	6.75 , 10.75		5.25 , 10.25
	CC	6.5 , 10.5		6.5 , 11.5
	RR	4 , 8		7 , 12

No choices for You or the Jury after this are strictly dominated. So the procedure ends here, after two steps.

3.2e) Elimination procedure Step 2

So MC, CC and RR are rational practice schedules for You under common belief in rationality. M+C and C+R are rational choices for the Jury under common belief in rationality.

3.2f) Updated epistemic model

3.2f) Updated epistemic model

Types	$T_Y = \{t_Y^{MC}, t_Y^{CC}, t_Y^{RR}\}$ $T_J = \{t_J^{M+C}, t_J^{C+R}\}$
Beliefs for You	$b_Y(t_Y^{MC}) = [M + C, t_J^{M+C}]$ $b_Y(t_Y^{CC}) = 0.5[M + C, t_J^{M+C}] + 0.5[C + R, t_J^{C+R}]$ $b_Y(t_Y^{RR}) = [C + R, t_J^{C+R}]$
Beliefs for Jury	$b_J(t_J^{M+C}) = [MC, t_Y^{MC}]$ $b_J(t_J^{C+R}) = [RR, t_Y^{RR}]$

Compared to 3.2a, we just delete those types of which we established that (a) were created for a choice that is never rational, or (b) did not express common belief in rationality.

Break

Let us take a break!

The Game: A new card game

- You propose a new card game to Barbara and Chris.
- From the deck of cards, there are 12 cards in game: Card numbers 2 to 10, and the Jack, Queen and King.
- Each card has a value. For the numbered cards the value is equal to the relevant number on the card. The Jack has value 11, the Queen has value 12 and the King has value 13.
- You receive the cards cards 3, 6, 9 and the Queen; Barbara receives the cards 2, 5, 8, and the Jack; Chris receives the cards 4, 7, 10 and the King.
- The allocation of cards is commonly known.
- Each player puts down a card of his or her choice on the table. The player that puts down the card with a value that is in the middle of all three cards on the table, wins.
- Every losing player has to pay up to the winner an amount equal to the value of the card he/she put down.

2.6a) Which cards are rational choices for you?

2.6a) Which cards are rational choices for you?

Every card could be rational for you, as every card could be the middle card for *some* combination of three cards on the table.

- Card 3:

2.6a) Which cards are rational choices for you?

Every card could be rational for you, as every card could be the middle card for *some* combination of three cards on the table.

- Card 3: $b_y = (2, 4)$; you believe Barbara chooses 2 and Chris 4.
- Card 6:

2.6a) Which cards are rational choices for you?

Every card could be rational for you, as every card could be the middle card for *some* combination of three cards on the table.

- Card 3: $b_y = (2, 4)$; you believe Barbara chooses 2 and Chris 4.
- Card 6: $b_y = (5, 7)$; Barbara chooses 5 and Chris chooses 7.
But also e.g. $b_y = (2, K)$.

2.6a) Which cards are rational choices for you?

Every card could be rational for you, as every card could be the middle card for *some* combination of three cards on the table.

- Card 3: $b_y = (2, 4)$; you believe Barbara chooses 2 and Chris 4.
- Card 6: $b_y = (5, 7)$; Barbara chooses 5 and Chris chooses 7.
But also e.g. $b_y = (2, K)$.
- Card 9: $b_y = (8, 10)$.
- Card Queen: $b_y = (J, K)$.

2.6b) Which cards are rational for Barbara and Chris?

2.6b) Which cards are rational for Barbara and Chris?

Let us start with Barbara. Barbara has the cards 2, 5, 8 and J in her hands.

2.6b) Which cards are rational for Barbara and Chris?

Let us start with Barbara. Barbara has the cards 2, 5, 8 and J in her hands. For Barbara all cards but card number 2 could possibly be a middle card.

- Card 5:

2.6b) Which cards are rational for Barbara and Chris?

Let us start with Barbara. Barbara has the cards 2, 5, 8 and J in her hands. For Barbara all cards but card number 2 could possibly be a middle card.

- Card 5: $b_B = (4, 6)$; Barbara believes Chris chooses 4 and that You choose 6.
- Card 8:

2.6b) Which cards are rational for Barbara and Chris?

Let us start with Barbara. Barbara has the cards 2, 5, 8 and J in her hands. For Barbara all cards but card number 2 could possibly be a middle card.

- Card 5: $b_B = (4, 6)$; Barbara believes Chris chooses 4 and that You choose 6.
- Card 8: $b_B = (7, 9)$; Chris chooses 7 and You choose 9.

2.6b) Which cards are rational for Barbara and Chris?

Let us start with Barbara. Barbara has the cards 2, 5, 8 and J in her hands. For Barbara all cards but card number 2 could possibly be a middle card.

- Card 5: $b_B = (4, 6)$; Barbara believes Chris chooses 4 and that You choose 6.
- Card 8: $b_B = (7, 9)$; Chris chooses 7 and You choose 9.
- Card J: $b_B = (10, Q)$.

2.6b) Which cards are rational for Barbara and Chris?

Let us start with Barbara. Barbara has the cards 2, 5, 8 and J in her hands. For Barbara all cards but card number 2 could possibly be a middle card.

- Card 5: $b_B = (4, 6)$; Barbara believes Chris chooses 4 and that You choose 6.
- Card 8: $b_B = (7, 9)$; Chris chooses 7 and You choose 9.
- Card J: $b_B = (10, Q)$.

Card number 2 is never the middle card. Does this mean it is irrational?

2.6b) Which cards are rational for Barbara and Chris?

Let us start with Barbara. Barbara has the cards 2, 5, 8 and J in her hands. For Barbara all cards but card number 2 could possibly be a middle card.

- Card 5: $b_B = (4, 6)$; Barbara believes Chris chooses 4 and that You choose 6.
- Card 8: $b_B = (7, 9)$; Chris chooses 7 and You choose 9.
- Card J: $b_B = (10, Q)$.

Card number 2 is never the middle card. Does this mean it is irrational? No, card number 2 can be rational if it is the **best losing option**.

2.6b) Which cards are rational for Barbara and Chris?

Let us start with Barbara. Barbara has the cards 2, 5, 8 and J in her hands. For Barbara all cards but card number 2 could possibly be a middle card.

- Card 5: $b_B = (4, 6)$; Barbara believes Chris chooses 4 and that You choose 6.
- Card 8: $b_B = (7, 9)$; Chris chooses 7 and You choose 9.
- Card J: $b_B = (10, Q)$.

Card number 2 is never the middle card. Does this mean it is irrational? No, card number 2 can be rational if it is the **best losing option**.

So if $b_B = (4, 3)$ (Chris chooses 4; You choose 3), then Barbara cannot do anything better than choosing 2.

2.6b) Which cards are rational for Barbara and Chris?

2.6b) Which cards are rational for Barbara and Chris?

Let us continue with Chris. Chris has the cards 4, 7, 10 and K in his hands.

2.6b) Which cards are rational for Barbara and Chris?

Let us continue with Chris. Chris has the cards 4, 7, 10 and K in his hands. For Chris all cards but the King could possibly be a middle card. So all cards but the King can *definitely* be rational for him.

- Card 4:

2.6b) Which cards are rational for Barbara and Chris?

Let us continue with Chris. Chris has the cards 4, 7, 10 and K in his hands. For Chris all cards but the King could possibly be a middle card. So all cards but the King can *definitely* be rational for him.

- Card 4: $b_C = (3, 5)$; Chris believes you choose 3 and Barbara chooses 5.
- Card 7:

2.6b) Which cards are rational for Barbara and Chris?

Let us continue with Chris. Chris has the cards 4, 7, 10 and K in his hands. For Chris all cards but the King could possibly be a middle card. So all cards but the King can *definitely* be rational for him.

- Card 4: $b_C = (3, 5)$; Chris believes you choose 3 and Barbara chooses 5.
- Card 7: $b_C = (6, 8)$; Chris believes you choose 6 and Barbara chooses 8.

2.6b) Which cards are rational for Barbara and Chris?

Let us continue with Chris. Chris has the cards 4, 7, 10 and K in his hands. For Chris all cards but the King could possibly be a middle card. So all cards but the King can *definitely* be rational for him.

- Card 4: $b_C = (3, 5)$; Chris believes you choose 3 and Barbara chooses 5.
- Card 7: $b_C = (6, 8)$; Chris believes you choose 6 and Barbara chooses 8.
- Card 10: $b_C = (9, J)$.

2.6b) Which cards are rational for Barbara and Chris?

Let us continue with Chris. Chris has the cards 4, 7, 10 and K in his hands. For Chris all cards but the King could possibly be a middle card. So all cards but the King can *definitely* be rational for him.

- Card 4: $b_C = (3, 5)$; Chris believes you choose 3 and Barbara chooses 5.
- Card 7: $b_C = (6, 8)$; Chris believes you choose 6 and Barbara chooses 8.
- Card 10: $b_C = (9, J)$.

The King is never the middle card. Does this mean it is irrational?

2.6b) Which cards are rational for Barbara and Chris?

Let us continue with Chris. Chris has the cards 4, 7, 10 and K in his hands. For Chris all cards but the King could possibly be a middle card. So all cards but the King can *definitely* be rational for him.

- Card 4: $b_C = (3, 5)$; Chris believes you choose 3 and Barbara chooses 5.
- Card 7: $b_C = (6, 8)$; Chris believes you choose 6 and Barbara chooses 8.
- Card 10: $b_C = (9, J)$.

The King is never the middle card. Does this mean it is irrational? Yes. It is never the middle card *and* it is never the best losing option. In fact, it is the worst losing option and **strictly dominated by any other card**.

2.6c) Beliefs diagram for the game

2.6c) Beliefs diagram for the game

See board

3.5a) Extended beliefs diagram and epistemic model

3.5a) Extended beliefs diagram and epistemic model

The extended beliefs diagram I will write on the board.

Types	$T_Y = \{t_Y^3, t_Y^6, t_Y^9, t_Y^Q\}$ $T_B = \{t_B^2, t_B^5, t_B^8, t_B^J\}$ $T_C = \{t_C^4, t_C^7, t_C^{10}, t_C^K\}$
Beliefs for You	$b_Y(t_Y^3) = [(2, t_B^2), (4, t_C^4)]$ $b_Y(t_Y^6) = [(5, t_B^5), (7, t_C^7)]$ $b_Y(t_Y^9) = [(8, t_B^8), (10, t_C^{10})]$ $b_Y(t_Y^Q) = [(J, t_B^J), (K, t_C^K)]$
Beliefs for Barbara	$b_B(t_B^2) = [(4, t_C^4), (3, t_Y^3)]$ $b_B(t_B^5) = [(4, t_C^4), (6, t_Y^6)]$ $b_B(t_B^8) = [(7, t_C^7), (9, t_Y^9)]$ $b_B(t_B^J) = [(10, t_C^{10}), (Q, t_Y^Q)]$
Beliefs for Chris	$b_C(t_C^4) = [(3, t_Y^3), (5, t_B^5)]$ $b_C(t_C^7) = [(6, t_Y^6), (8, t_B^8)]$ $b_C(t_C^{10}) = [(9, t_Y^9), (J, t_B^J)]$ $b_C(t_C^K) = [(Q, t_Y^Q), (J, t_B^J)]$

3.5b) Writing down third-order beliefs for type t_Y^Q

- First level:

3.5b) Writing down third-order beliefs for type t_Y^Q

- First level: You believe that (1) Barbara will choose the Jack and that Chris will choose the King.
- Second level:

3.5b) Writing down third-order beliefs for type t_Y^Q

- First level: You believe that (1) Barbara will choose the Jack and that Chris will choose the King.
- Second level:
 - You believe that (2B) Barbara believes that Chris will choose 10 and that You will choose Q ;

3.5b) Writing down third-order beliefs for type t_Y^Q

- First level: You believe that (1) Barbara will choose the Jack and that Chris will choose the King.
- Second level:
 - You believe that (2B) Barbara believes that Chris will choose 10 and that You will choose Q ;
 - You also believe that (2C) Chris believes you will choose Q and that Barbara will choose J .
- Third level:

3.5b) Writing down third-order beliefs for type t_Y^Q

- First level: You believe that (1) Barbara will choose the Jack and that Chris will choose the King.
- Second level:
 - You believe that (2B) Barbara believes that Chris will choose 10 and that You will choose Q ;
 - You also believe that (2C) Chris believes you will choose Q and that Barbara will choose J .
- Third level:
 - You believe that (3BY) Barbara believes that You believe that Barbara will choose J and Chris will choose K ;
 - You believe that (3BC) Barbara believes that Chris believes that You will choose 9 and Barbara will choose J ;
 - You believe that (3CY) that Chris believes that You believe that Barbara will choose J and Chris K ;
 - You believe that (3CB) that Chris believes that Barbara believes that Chris will choose 10 and that You will choose Q .

3.5b) Writing down third-order beliefs for type t_Y^6

- First level:

3.5b) Writing down third-order beliefs for type t_Y^6

- First level: You believe that (1) Barbara will choose 5 and that Chris will choose 7.
- Second level:
 - You believe that (2B) Barbara believes that Chris will choose 4 and that You will choose 6;
 - You also believe that (2C) Chris believes you will choose 6 and that Barbara will choose 8.
- Third level:
 - You believe that (3BY) Barbara believes that You believe that Barbara will choose 5 and Chris will choose 7;
 - You believe that (3BC) Barbara believes that Chris believes that You will choose 3 and Barbara will choose 5;
 - You believe that (3CY) that Chris believes that You believe that Barbara will choose 5 and Chris 7;
 - You believe that (3CB) that Chris believes that Barbara believes that Chris will choose 7 and that You will choose 9.

3.5c) Which cards can you and Barbara and Chris choose under CBR?-graphical method

See board.

3.5c) Which cards can you and Barbara and Chris choose under CBR?-graphical method

See board. In any case, no type in the epistemic model expresses CBR. At some point each arrow in the beliefs diagram will pass by the King-card for Chris, which is never rational.

The important thing here to notice is that, wherever you choose to start in the diagram, eventually you *will* pass by the King-card in your reasoning. The King card is never a rational choice. Hence no type for any person in the model expresses common belief in rationality. But we can change the diagram in such a way such that CBR is present (See also question e) later).

3.5d) Which cards can you and Barbara and Chris choose under CBR?-elimination procedure

3.5d) Which cards can you and Barbara and Chris choose under CBR?-elimination procedure

Iterated elimination of strictly dominated choice: at each round, eliminate all strictly dominated choices in the (reduced) decision problem.

- Round 1:

3.5d) Which cards can you and Barbara and Chris choose under CBR?-elimination procedure

Iterated elimination of strictly dominated choice: at each round, eliminate all strictly dominated choices in the (reduced) decision problem.

- Round 1: Eliminate **King** for Chris. As we saw, King was strictly dominated by e.g. card number 10

3.5d) Which cards can you and Barbara and Chris choose under CBR?-elimination procedure

Iterated elimination of strictly dominated choice: at each round, eliminate all strictly dominated choices in the (reduced) decision problem.

- Round 1: Eliminate **King** for Chris. As we saw, King was strictly dominated by e.g. card number 10
- Round 2:

3.5d) Which cards can you and Barbara and Chris choose under CBR?-elimination procedure

Iterated elimination of strictly dominated choice: at each round, eliminate all strictly dominated choices in the (reduced) decision problem.

- Round 1: Eliminate **King** for Chris. As we saw, King was strictly dominated by e.g. card number 10
- Round 2: Eliminate **Queen** for You.

3.5d) Which cards can you and Barbara and Chris choose under CBR?-elimination procedure

Iterated elimination of strictly dominated choice: at each round, eliminate all strictly dominated choices in the (reduced) decision problem.

- Round 1: Eliminate **King** for Chris. As we saw, King was strictly dominated by e.g. card number 10
- Round 2: Eliminate **Queen** for You. You face a new decision problem, which does not include the King for Chris. So the Queen will never be the middle card, and it is for you the worst losing option left in the problem.
- Round 3:

3.5d) Which cards can you and Barbara and Chris choose under CBR?-elimination procedure

Iterated elimination of strictly dominated choice: at each round, eliminate all strictly dominated choices in the (reduced) decision problem.

- Round 1: Eliminate **King** for Chris. As we saw, King was strictly dominated by e.g. card number 10
- Round 2: Eliminate **Queen** for You. You face a new decision problem, which does not include the King for Chris. So the Queen will never be the middle card, and it is for you the worst losing option left in the problem.
- Round 3: Eliminate **Jack** for Barbara.

3.5d) Which cards can you and Barbara and Chris choose under CBR?-elimination procedure

Iterated elimination of strictly dominated choice: at each round, eliminate all strictly dominated choices in the (reduced) decision problem.

- Round 1: Eliminate **King** for Chris. As we saw, King was strictly dominated by e.g. card number 10
- Round 2: Eliminate **Queen** for You. You face a new decision problem, which does not include the King for Chris. So the Queen will never be the middle card, and it is for you the worst losing option left in the problem.
- Round 3: Eliminate **Jack** for Barbara. Barbara faces a new decision problem, which does not include the King for Chris or the Queen for You. So the Jack will never be the middle card. Moreover it is the worst losing option left for Barbara.

3.5d) Which cards can you and Barbara and Chris choose under CBR?-elimination procedure

Iterated elimination of strictly dominated choice: at each round, eliminate all strictly dominated choices in the (reduced) decision problem.

- Round 1: Eliminate **King** for Chris. As we saw, King was strictly dominated by e.g. card number 10
- Round 2: Eliminate **Queen** for You. You face a new decision problem, which does not include the King for Chris. So the Queen will never be the middle card, and it is for you the worst losing option left in the problem.
- Round 3: Eliminate **Jack** for Barbara. Barbara faces a new decision problem, which does not include the King for Chris or the Queen for You. So the Jack will never be the middle card. Moreover it is the worst losing option left for Barbara.

3.5d) Which cards can you and Barbara and Chris choose under CBR?-elimination procedure

- Round 4:

3.5d) Which cards can you and Barbara and Chris choose under CBR?-elimination procedure

- Round 4: Eliminate 10 for Chris.

3.5d) Which cards can you and Barbara and Chris choose under CBR?-elimination procedure

- Round 4: Eliminate 10 for Chris. Chris faces a new decision problem, which does not include the King, the Queen or the Jack. The 10 is the highest card left, and will thus never be the middle card. It is also the worst losing option left for Chris at this point.

3.5d) Which cards can you and Barbara and Chris choose under CBR?-elimination procedure

- Round 4: Eliminate 10 for Chris. Chris faces a new decision problem, which does not include the King, the Queen or the Jack. The 10 is the highest card left, and will thus never be the middle card. It is also the worst losing option left for Chris at this point.
- In Round 5 we eliminate 9 for You; in Round 6 we eliminate 8 for Barbara; in Round 7 we eliminate 7 for Chris; in Round 8 we eliminate 6 for you; in Round 9 we eliminate 5 for Barbara.
- There are no more choices to eliminate for any player \Rightarrow We are left with 2 for Barbara, 3 for You and 4 for Chris.

3.5e) New epistemic model that expresses CBR

We need to create a new epistemic model. For each choice identified in (d) we have to create a type, and this type should be such that it optimizes the relevant choice *and* expresses common belief in rationality.

3.5e) New epistemic model that expresses CBR

We need to create a new epistemic model. For each choice identified in (d) we have to create a type, and this type should be such that it optimizes the relevant choice *and* expresses common belief in rationality.

	$T_Y = \{t_Y^3\}$
Types	$T_B = \{t_B^2\}$
	$T_C = \{t_C^4\}$
Beliefs for You	$b_Y(t_Y^3) = [(2, t_B^2), (4, t_C^4)]$
Beliefs for Barbara	$b_B(t_B^2) = [(4, t_C^4), (3, t_Y^3)]$
Beliefs for Chris	$b_C(t_C^4) = [(3, t_Y^3), (2, t_B^2)]$

3.5f) Who is the winner?

3.5f) Who is the winner?

Types	$T_Y = \{t_Y^3\}$ $T_B = \{t_B^2\}$ $T_C = \{t_C^4\}$
Beliefs for You	$b_Y(t_Y^3) = [(2, t_B^2), (4, t_C^4)]$
Beliefs for Barbara	$b_B(t_B^2) = [(4, t_C^4), (3, t_Y^3)]$
Beliefs for Chris	$b_C(t_C^4) = [(3, t_Y^3), (2, t_B^2)]$

3.5f) Who is the winner?

Types	$T_Y = \{t_Y^3\}$ $T_B = \{t_B^2\}$ $T_C = \{t_C^4\}$
Beliefs for You	$b_Y(t_Y^3) = [(2, t_B^2), (4, t_C^4)]$
Beliefs for Barbara	$b_B(t_B^2) = [(4, t_C^4), (3, t_Y^3)]$
Beliefs for Chris	$b_C(t_C^4) = [(3, t_Y^3), (2, t_B^2)]$

- We see that You win this game if all players reason in line with common belief in rationality.
- Barbara and Chris choose their 'best losing option': They pay the minimum amount you can pay as a loser.
- You win $2 + 4 = 6$ euros.