

EpiCenter Spring Course on Epistemic Game Theory



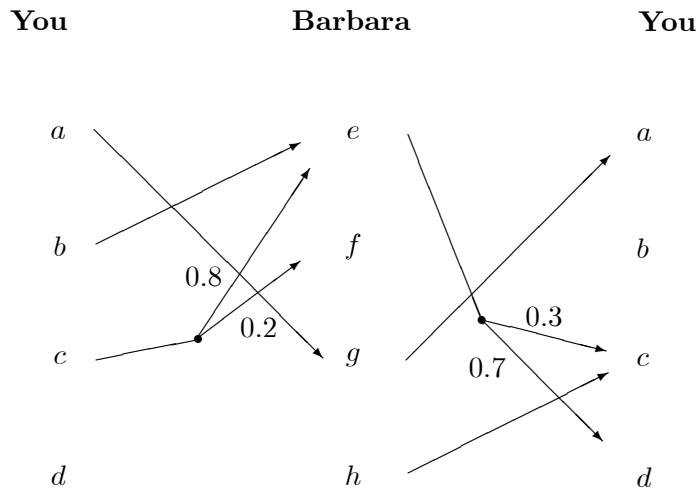
Maastricht University

June 8 – 19, 2015
Exam

Problem 1 (Standard beliefs in static games): 40 points

For each part you can score 5 points.

- (a) Give the definition of an epistemic model with standard beliefs.
- (b) Explain, both verbally and formally, what it means for a type to express common belief in rationality.
- (c) Suppose that within a given epistemic model, all types believe in the opponents' rationality. Explain why in this case, all types within the epistemic model also express *common* belief in rationality.
- (d) What does it mean for a type to have a *simple belief hierarchy*?
- (e) What is the formal connection between common belief in rationality, simple belief hierarchies, and Nash equilibrium? Which conceptual problems with Nash equilibrium does epistemic game theory reveal?
- (f) Consider the following beliefs diagram.



Translate this beliefs diagram into an epistemic model.

(**Hint:** For those choices that do not have an outgoing arrow, you may construct a type with an arbitrary belief).

- (g) Which of the types in your epistemic model express common belief in rationality? Explain.
- (h) Which of the types in your epistemic model hold a simple belief hierarchy? Explain.

Problem 2 (Lexicographic beliefs in static games): 30 points

(a) (5 points) Explain the difference between primary belief in the opponent's rationality, respecting the opponent's preferences, and assuming the opponent's rationality.

Story: A colleague at work wants to paint his living room, but he has two left hands. Therefore, he wants to ask somebody else to do the job. Both you and your friend Barbara have told him to be interested in the task. In order to decide which person will get the job, he invites you and Barbara to his favorite pub, where he proposes the following procedure: Barbara and you must simultaneously write down a price on a piece of paper. The price must be either 100 euros, or 200 euros, or 300 euros. The person who has written down the lowest price will get the job, and will be paid exactly that amount. In case you both write down the same price, your colleague will choose Barbara, since she knows the colleague longer than you do.

(b) (3 points) Formulate this situation as a game between Barbara and you.

(c) (2 points) Which prices can you rationally write down under common belief in rationality with standard beliefs? What about Barbara? Explain.

(d) (3 points) Which prices can you rationally write down under common full belief in "caution and primary belief in rationality"? What about Barbara? What procedure do you use?

(e) (7 points) Which prices can you rationally write down under common full belief in "caution and respect of preferences"? What about Barbara? What procedure do you use?

(f) (7 points) Construct an epistemic model with lexicographic beliefs such that

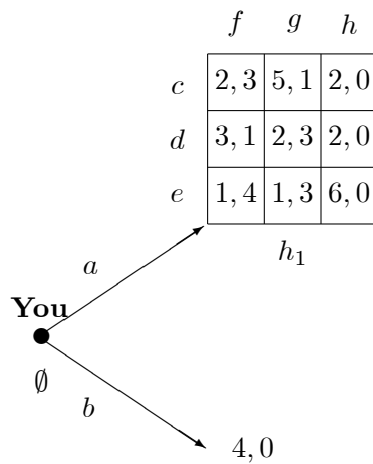
- every type in this model is cautious and expresses common full belief in "caution and respect of preferences", and
- for every choice found in (e) there is a type in this model for which this choice is optimal.

(g) (3 points) Which prices can you rationally write down under common assumption of rationality? What about Barbara? What procedure do you use?

Problem 3 (Dynamic games): 30 points

(a) (5 points) Explain the difference between belief in the opponents' future rationality and strong belief in the opponents' rationality.

Consider the dynamic game below between you and Barbara. If you choose a at the beginning, then you can choose between c, d and e , whereas Barbara can choose between f, g and h .



(b) (7 points) Which strategies can you rationally choose under common belief in future rationality? What about Barbara? What procedure do you use?

(c) (7 points) Construct an epistemic model such that, for every strategy found in (b), there is some type that expresses common belief in future rationality, and for which that strategy is rational.

(d) (7 points) Which strategies can you rationally choose under common strong belief in rationality? What about Barbara? What procedure do you use?

(e) (4 points) Explain intuitively why you find different results in (b) and (d).