

EPICENTER Spring Course in Epistemic Game Theory

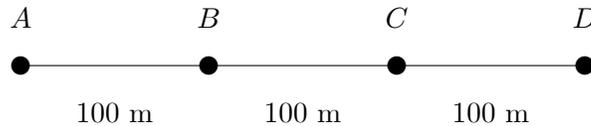


Maastricht University, June 12–26, 2017
Exam

June 26, 9.00–12.00, Room A 0.23

Problem 1: To dance or not to dance (40 points)

This evening, both Barbara and you feel like dancing. In Maastricht there are four places which are perfect for dancing: Amorsplein, Brandweerkantine, Charlemagne and D'n Ingel. Below you find a map of these four places, where A stands for Amorsplein, B for Brandweerkantine, C for Charlemagne and D for D'n Ingel.



As you can see on the map, the distance between the two closest dancing places is always 100 meters.

Both you and Barbara can choose between studying at home for the epistemic game theory exam, or going to one of these four places for an endless evening of dancing, not knowing where the other person will go. The problem, however, is that you are a terrible dancer whereas Barbara is a real dancing queen. For that reason, you would like to be as close as possible to Barbara when you go out dancing, whereas Barbara would like to be as far away from you as possible.

More precisely, if both you and Barbara go out dancing, then your utility would be 300 minus the distance (in meters) between the two places you and Barbara go to. Barbara's utility, on the other hand, would be exactly the distance (in meters) between these two places.

If you stay at home and study for the exam, your utility will be 250. However, if you go out dancing and Barbara stays at home studying for the exam, then you feel guilty and your utility will be 0. The same holds for Barbara.

- (a) (3 points) Model this situation as a game between you and Barbara. That is, list the possible choices for you and Barbara, and the utilities for each combination of choices.
- (b) (7 points) Find all the choices that you and Barbara can rationally make under common belief in rationality. Which algorithm do you use?
- (c) (4 points) Construct a beliefs diagram for this game.
- (d) (6 points) Construct an epistemic model for this game such that for every choice c_i found in (b) there is a type t_i in the epistemic model that expresses common belief in rationality, and for which c_i is optimal.
- (e) (4 points) Which types in your epistemic model have a simple belief hierarchy, and which do not? Explain.
- (f) (10 points) Find all the choices that you and Barbara can rationally make under common belief in rationality with a simple belief hierarchy.
- (g) (6 points) Discuss the conceptual problems with the concept of Nash equilibrium, both in general and in this specific game.

Problem 2: A night at the movies (30 points)

This evening, Barbara and you will both go to the same cinema to watch *The Godfather Part I*. You will both order your tickets online, and the only seats left are the ones displayed below:

1	2	3	4	5	6	7
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As you can see, all seats are in the same row, next to each other. The task for you and Barbara is to select one of these seven seats. If you happen to select the same seat, then the computer system will automatically select two seats next to each other for you and Barbara.

However, yesterday you happened to go to the same dancing place as Barbara, embarrassing her in front of all her friends with your terrible dancing skills. Not surprisingly, Barbara is now angry at you, and would like to avoid sitting next to you at the movies. Since you would like to avoid her angry face, you also would prefer not to sit next to Barbara. More precisely, if you happen to sit next to each other then the utility for both you and Barbara will be 0. If you do not sit next to each other, then the utility for both you and Barbara will be 1.

(a) (3 points) Which choices can you rationally make under common belief in rationality with standard beliefs? Explain.

(b) (5 points) Find all the choices you can rationally make under common full belief in “caution and primary belief in rationality”. Which algorithm do you use?

(c) (7 points) Find all the choices you can rationally make under 1-fold assumption of rationality, under 1-fold and 2-fold assumption of rationality, and under common assumption of rationality. Which algorithm do you use?

(d) (8 points) Construct an epistemic model such that for every choice c_i that can rationally be made under common assumption of rationality, there is a type t_i that expresses common assumption of rationality and for which c_i is optimal.

(e) (7 points) Consider the epistemic model where there is one type t_1 for you, one type t_2 for Barbara, and where the lexicographic beliefs of the types are given by

$$\begin{aligned} b_1(t_1) &= ((7, t_2); (6, t_2); (5, t_2); (4, t_2); (3, t_2); (2, t_2); (1, t_2)) \\ b_2(t_2) &= ((1, t_1); (2, t_1); (3, t_1); (4, t_1); (5, t_1); (6, t_1); (7, t_1)). \end{aligned}$$

Does your type t_1 primarily believe in Barbara’s rationality? Does t_1 assume Barbara’s rationality? Does t_1 respect Barbara’s preferences? Explain.

Problem 3: Champions League final 2018, 92nd minute (30 points)

You are a famous football player, playing the Champions League final 2018 against Bayern Munich. The score is 2-2, it is the 92nd minute, you are heading towards Bayern's goalkeeper Neuer, and then suddenly you are kicked from behind. At this moment you have two options: you can either let yourself fall and get a free kick, or continue towards Bayern's goal and shoot. If you continue, then you must choose between shooting to the left corner or shooting to the right corner of the goal (from your perspective). At the same time, Neuer must decide between diving to the left corner or diving to the right corner (from your perspective).

If Neuer dives to the same corner where you shoot the ball, he will always save the ball, as he is such a good keeper. If you shoot to the left and Neuer dives to the right, you will score with probability 70% only, as there is a 30% chance that you will miss the goal in this case. If you shoot to the right and Neuer dives to the left, things are even worse since you will only score with probability 40% then. Hence, you are better at shooting to the left corner than shooting to the right corner. Finally, if you let yourself fall and get a free kick, you will score with probability 50%, as you are a specialist in free kicks.

Obviously, your objective in this game is to maximize the probability of scoring, and the objective of Neuer is to maximize the probability of you *not* scoring.

- (a) (3 points) Model this situation as a dynamic game between you and goalkeeper Neuer.
- (b) (3 points) Explain intuitively the difference between “belief in future rationality” and “strong belief in rationality”.
- (c) (4 points) Find all the strategies you can rationally choose under common belief in future rationality? Which algorithm do you use?
- (d) (7 points) Find all the strategies you can rationally choose under common strong belief in rationality? Which algorithm do you use?
- (e) (3 points) Describe the intuitive reasoning that leads to the strategy choices in (d).
- (f) (7 points) Construct an epistemic model such that for every strategy s_i found in (d) there is a type t_i that expresses common strong belief in rationality and for which s_i is rational. Which types in your epistemic model strongly believe in the opponent's rationality? Which types express common strong belief in rationality?
- (g) (3 points) Can you construct an epistemic model for this game where *all* types express common strong belief in rationality? Explain.