Utility Proportional Beliefs

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Outline

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Asymmetric Matching Pennies

In this asymmetric matching pennies game Nash equilibrium would give a quite unintuitive prediction.

		Column Player	
		left	*right*
Row Player	*top*	320,40	40,80
	bottom	40,80	80,40

Asymmetric Matching Pennies

In this asymmetric matching pennies game Nash equilibrium would give a quite unintuitive prediction.

Column Player

		left	*right*	
Row Player	*top*	320,40	40,80	*50%*
	bottom	40,80	80,40	*50%*
		12.5%	*87.5%*	

The percentages indicate the outcome of the mixed nash equilibrium.

Asymmetric Matching Pennies

In this asymmetric matching pennies game Nash equilibrium would give a quite unintuitive prediction.

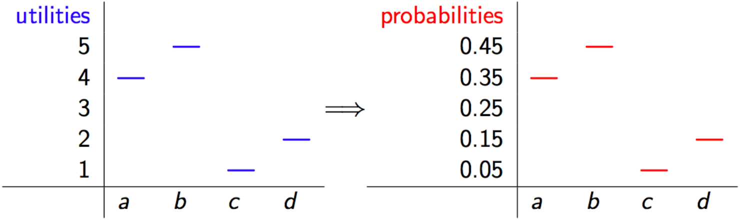
Column Player

Row Player		*left*	*right*	
	top	320,40	40,80	*50% (96%)*
	bottom	40,80	80,40	*50% (4%)*
		12.5% (16%)	*87.5% (84%)*	

The percentages indicate the outcome of the mixed nash equilibrium. In parentheses are the resuluts that [?] found in their experiment.

Utility Proportional Beliefs [?]

A solution concept in which players form their belief proportional to the differences in their opponents' utilities.



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- Allows individual differences in sensitivity to differences in utility
- Invariant to affine transformations

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Epistemic Model

In the following we will restrict our view to the two player case. Note that these definitions can easily be extended to N players.

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- $u_i : C_1 \times C_2 \to \mathbb{R}$

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Definition

An epistemic model M for Γ specifies

1. for all players i a set of types T_i, and
2. for every type $t_i \in T_i$ some probabilistic belief $b_i(t_i)$ on $C_j \times T_j$ with finite support.

Operationalization of Utility Proportional Beliefs

Let $i, j \in I$ be the two players, and $\lambda_j \in \mathbb{R}$ be such that $\lambda_j \geq 0$.

Definition

A type $t_i \in T_i$ of player i expresses λ_j-utility-proportional-beliefs, if

$$(b_i(t_i))(c_j|t_j) - (b_i(t_i))(c_j'|t_j) = \frac{\lambda_j}{\overline{u}_j - \underline{u}_j}(u_j(c_j, t_j) - u_j(c_j', t_j))$$

for all $t_j \in T_j(t_i)$, *for all* c_j, $c_j' \in C_j$.

Invariance to Affine Transformations

$$
\begin{aligned}
(b_i(t_i))(c_j|t_j) - (b_i(t_i))(c_j'|t_j) &= \frac{\lambda_j}{\bar{\hat{u}}_j - \underline{\hat{u}}_j}(\hat{u}_j(c_j, t_j) - \hat{u}_j(c_j', t_j)) \\
&= \frac{\lambda_j}{(a + b\bar{u}_j) - (a + b\underline{u}_j)}((a + bu_j(c_j, t_j)) - (a + bu_j(c_j', t_j))) \\
&= \frac{\lambda_j}{b\bar{u}_j - b\underline{u}_j}(bu_j(c_j, t_j) - bu_j(c_j', t_j)) \\
&= \frac{\lambda_j}{b(\bar{u}_j - \underline{u}_j)} b(u_j(c_j, t_j) - u_j(c_j', t_j)) \\
&= \frac{\lambda_j}{\bar{u}_j - \underline{u}_j}(u_j(c_j, t_j) - u_j(c_j', t_j))
\end{aligned}
$$

Common Belief in λ-Utility-Proportional-Beliefs

Let $i, j \in I$ be the two players, and $\lambda_j \in \mathbb{R}$.

Lemma

A type $t_i \in T_i$ of player i expresses λ_j-utility-proportional-beliefs if and only if

$$(b_i(t_i))(c_j|t_j) = \frac{1}{|C_j|} + \frac{\lambda_j}{\bar{u}_j - \underline{u}_j}(u_j(c_j, t_j) - u_j^{average}(t_j)),$$

for all $t_j \in T_j(t_i)$, for all $c_j \in C_j$, and for $j \in I \backslash \{i\}$.

Define λ_j^{max} such that $(b_i(t_i))(\cdot|t_j)$ yields a well-defined probability measure for every type $t_j \in T_j$.

Utility Proportional Beliefs

$$
\begin{aligned}
1 &= \sum_{c_j \in C_j} (b_i(t_i))(c_j|t_j) \\
&= \sum_{c_j \in C_j} \left((b_i(t_i))(c_j^*|t_j) + (b_i(t_i))(c_j|t_j) - (b_i(t_i))(c_j^*|t_j)\right) \\
&= \sum_{c_j \in C_j} \left((b_i(t_i))(c_j^*|t_j) + \frac{\lambda_j}{\bar{\hat{u}}_j - \underline{\hat{u}}_j}(u_j(c_j, t_j) - u_j(c_j^*, t_j))\right) \\
&= |C_j|(b_i(t_i))(c_j^*|t_j) + \frac{\lambda_j}{\bar{\hat{u}}_j - \underline{\hat{u}}_j} \sum_{c_j \in C_j} (u_j(c_j, t_j) - u_j(c_j^*, t_j)) \\
&= |C_j|(b_i(t_i))(c_j^*|t_j) + \frac{\lambda_j}{\bar{\hat{u}}_j - \underline{\hat{u}}_j} \left(|C_j| u_j^{average}(c_j, t_j) - |C_j| u_j(c_j^*, t_j)\right)
\end{aligned}
$$

$$
(b_i(t_i))(c_j^*|t_j) = \frac{1}{|C_j|} + \frac{\lambda_j}{\bar{\hat{u}}_j - \underline{\hat{u}}_j} \left(u_j(c_j^*, t_j) - u_j^{average}(c_j, t_j)\right)
$$

Definition

Let $i, j \in I$ be the two players and $t_i \in T_i$ be some type of player i, and $\lambda = (\lambda_i)_{i \in I} \in \times_{i \in I} \mathbb{R}$.

Definition

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- *Type t_i expresses 1-fold belief in λ-utility-proportional-beliefs, if t_i expresses λ_i-utility-proportional-beliefs.*

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- *Type t_i expresses 1-fold belief in λ-utility-proportional-beliefs, if t_i expresses λ_i-utility-proportional-beliefs.*
- *Type t_i expresses k-fold belief in λ-utility-proportional-beliefs, if $(b_i(t_i))$ only deems possible types $t_j \in T_j$ for j such that t_j expresses $k-1$-fold belief in λ-utility-proportional-beliefs, for all $k > 1$.*

Definition

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- *Type t_i expresses common belief in λ-utility-proportional-beliefs, if t_i expresses k-fold belief in λ-utility-proportional-beliefs for all $k \geq 1$.*

Rationality can be defined the following way:

Definition

Let $i, j \in I$ be the two players and $t_i \in T_i$ be some type of player i, and $\lambda = (\lambda_i)_{i \in I} \in \times_{i \in I}\mathbb{R}$. *A choice* $c_i \in C_i$ *of player* i *is rational under common belief in* λ-utility-proportional-beliefs, *if there exists an epistemic model* $\mathcal{M}^\Gamma$ *and some type* $t_i \in T_i$ *of player* i *such that* c_i *is optimal given* $(b_i(t_i))$ *and* t_i *expresses common belief in* λ-utility-proportional-beliefs.

Algorithm

For player i and her opponent $j \in I \setminus \{i\}$, $p_i^* : \Delta(C_i) \to \Delta(C_j)$, where p_i^* is a function mapping the beliefs of player j on her opponent's choice combinations to beliefs on j's choice

$$(p_i^*(p_j))(c_j) := \frac{1}{|C_j|} + \frac{\lambda_j}{\overline{u}_j - \underline{u}_j}(u_j(c_j, p_j) - u_j^{average}(p_j))$$

for all $c_j \in C_j$ and for all $p_j \in P_j^0$, where $P_j^0 := \Delta(C_i)$.

Algorithm

Definition

For both players $i, j \in I$ and for all $k \geq 0$ the set P_i^k of i's beliefs about her opponent's choice combinations is inductively defined as follows:

$$
\begin{aligned}
P_i^0 &:= \Delta(C_j), \text{ and} \\
P_i^k &:= p_i^*(P_j^{k-1}).
\end{aligned}
$$

The set of belief $P_i^\infty = \cap_{k \geq 0} P_i^k$ contains the beliefs that survive iterated elimination of utility-disproportional-beliefs.

First we will show that the beliefs surviving the elminiation procedure are unique, and then that this algorithm selects exactly those beliefs that players can holder under common believe in λ-utility-proportional beliefs.

Recall: λ_i^{max} is the largest proportionality factor such that $p_i^*(p_j, \lambda_i)$ is a probability distribution for all $p_j \in P_j^0$.

Note: Whenever $\lambda_i < \lambda_i^{max}$ UPB always assigns positive probability to all of j's choices.

Definition

$\alpha A + (1-\alpha)B := \{\alpha a + (1-\alpha)b : a \in A, b \in B\}$ for $A, B \in P_j^0$.

Definition

$\alpha_i := \frac{\lambda_i}{\lambda_i^{max}} < 1$ and $\alpha := \max\{\alpha_i, \alpha_j\}$.

Lemma

For every player $i \in I$ and every round $k \geq 0$ there exists $p_i \in P_i^0$ such that $P_i^k \subseteq \alpha^k P_i^0 + (1 - \alpha^k)\{p_i\}$.

Proof

We prove the statement by induction on k.

1) Let $k = 0$, then $P_i^0 \subseteq \alpha^0 P_i^0 + (1 - \alpha^0)\{p_i\}$ holds for all $p_i \in P_i^0$ because $\alpha^0 = 1$.

2) Suppose $k > 0$ and assume that there exists $p_i \in P_i^0$ such that $P_i^{k-1} \subseteq \alpha^{k-1} P_i^0 + (1-\alpha^{k-1})\{p_i\}$.

Note: $P_i^k = p_i^*(P_j^{k-1}, \lambda_i)$ by the construction of the algorithm.

Then,

$$\begin{aligned} P_i^k &\subseteq p_i^*(\alpha^{k-1} P_j^0 + (1-\alpha^{k-1})\{p_j\}, \lambda_i) \\ &= \alpha^{k-1} p_i^*(P_j^0) + (1-\alpha^{k-1})\{p_i^*(p_j)\} \end{aligned}$$

by linearity of p_i^* on P_j^0.

By definition of α_i, $\lambda_i = \alpha_i \lambda_i^{max} + (1 - \alpha_i)0$.

As p_i^* is also linear in λ_i, it follow that

$$p_i^*(P_j^0, \lambda_i) = \alpha_i p_i^*(P_j^0, \lambda_i^{max}) + (1 - \alpha_i) p_i^*(P_j^0, 0),$$

where $p_i(P_j^0, 0) = p_i^{uni}$.

Since also $p_i^*(P_j^0, \lambda_i^{max}) \subseteq P_i^0$, it holds that

$$p_i^*(P_j^0, \lambda_i) \subseteq \alpha_i P_i^0 + (1 - \alpha_i)\{p_i^{uni}\}.$$

As $\alpha_i \leq \alpha$ we have

$$\alpha_i P_i^0 + (1 - \alpha_i)\{p_i^{uni}\} \subseteq \alpha P_i^0 + (1 - \alpha)\{p_i^{uni}\}$$

and hence $p_i^*(P_j^0, \lambda_i) \subseteq \alpha P_i^0 + (1 - \alpha)\{p_i^{uni}\}$.

Combining the two previous results:

$$\begin{aligned} P_i^k &\subseteq \alpha^{k-1}\left(\alpha P_i^0 + (1-\alpha)\{p_i^{uni}\}\right) + (1-\alpha^{k-1})\{p_i^*(p_j,\lambda_i)\} \\ &= \alpha^k P_i^0 + \alpha^{k-1}(1-\alpha)\{p_i^{uni}\} + (1-\alpha^{k-1})\{p_i^*(p_j,\lambda_i)\}. \end{aligned}$$

Recall that $\alpha < 1$, then by defining

$$\hat{p}_i := \frac{\alpha^{k-1}(1-\alpha)p_i^{uni} + (1-\alpha^{k-1})p_i^*(p_j,\lambda_i)}{1-\alpha^k},$$

it follows that $P_i^k \subseteq \alpha^k P_i^0 + (1-\alpha^k)\{\hat{p}_i\}$.

Asymmetric Matching Pennies

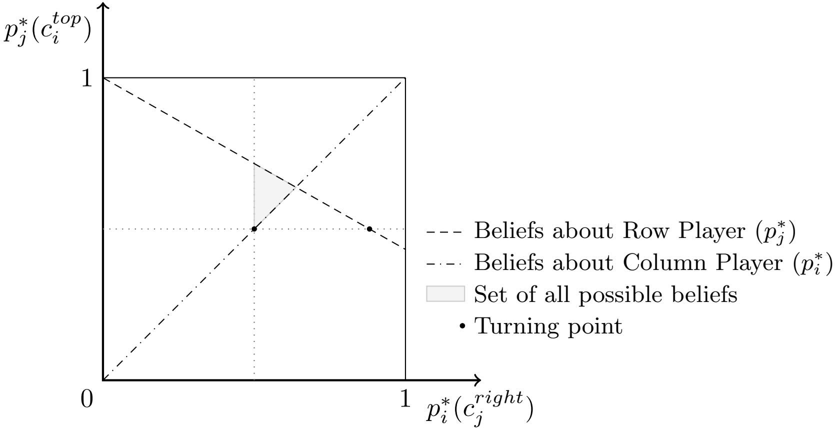
Let's compute UPB for our example:

		Column Player	
		left	*right*
Row Player	*top*	320,40	40,80
	bottom	40,80	80,40

Fixed Point

We observe that the convex combination converges to a singleton and hence to a unique fixed point. Now suppose $p_i' \in P_i^{\infty}$ is the belief of i and $p_j' \in P_j^{\infty}$ is the belief of j for which equation $P_i^k \subseteq \alpha^k P_i^0 + (1-\alpha^k)\{p_i'\}$ holds.

Recall, that by the construction of the algorithm it holds that $P_i^k = p_i^*(P_j^{k-1}, \lambda_j)$. Then it holds that $p_i^*(p_j') = p_i'$ and $p_j^*(p_i') = p_j'$ and hence it also holds that $p_i^*(p_j^*(p_i')) = p_i'$. Consequently, the mapping $p_i^* \circ p_j^*$ has a unique fixed point in two-player games where the players' are holding common belief in λ-*utility-proportional-beliefs*.


$p_j^*(c_i^{top})$
1
0
1
$p_i^*(c_j^{right})$
Beliefs about Row Player (p_j^*)
Beliefs about Column Player (p_i^*)
Set of all possible beliefs
Turning point

Theorem

Let $\lambda = (\lambda_i)_{i \in I} \in \times_{i \in I} \mathbb{R}_+$ such that $\lambda_i < \lambda_i^{max}$ for all players $i \in I$. A belief $p_i \in \Delta(C_j)$ can be held by a type $t_i \in T_i$ that expresses common belief in λ-utility-proportional beliefs in some epistemic model $\mathcal{M}^\Gamma$ of Γ if and only if p_i survives iterated elimination of utility-disproportional beliefs.

Proof

First of all, let $k = 1$ and consider $t_i \in T_i$ that expresses 1-fold belief in λ-utility-proportional beliefs. Then,

$$(b_i(t_i))(c_j, t_j) = \left(\frac{1}{|C_j|} + \frac{\lambda_j}{\bar{u}_j - \underline{u}_j} (u_j(c_j, t_j) - u_j^{average}(t_j)) \right) (b_i(t_i))(t_j)$$

for all $c_j \in C_j$ and $t_j \in T_j$. It follows that

$$(b_i(t_i))(c_j) = \sum_{t_j \in T_j} (b_i(t_i))(t_j) \left(\frac{1}{|C_j|} + \frac{\lambda_j}{\bar{u}_j - \underline{u}_j} (u_j(c_j, t_j) - u_j^{average}(t_j)) \right)$$

for all $c_j \in C_j$.

Written as vector

$$(b_i(t_i))(c_j)_{c_j \in C_j} =$$
$$\sum_{t_j \in T_j} (b_i(t_i))(t_j) \left(\frac{1}{|C_j|}(1, \ldots, 1) + \frac{\lambda_j}{\bar{u}_j - \underline{u}_j}(u_j(c_j, t_j) - u_j^{average}(t_j))_{c_j \in C_j} \right).$$

Note that by definition,

$$\frac{1}{|C_j|}(1, \ldots, 1) + \frac{\lambda_j}{\bar{u}_j - \underline{u}_j}(u_j(c_j, t_j) - u_j^{average}(t_j))_{c_j \in C_j} \in P_i^1.$$

Since $(b_i(t_i))(c_j)_{c_j \in C_j}$ is a convex combination of elements in the convex set P_i^1, it follows that $(b_i(t_i))(c_j)_{c_j \in C_j} \in P_i^1$.

Now let $k > 1$ and consider $t_i \in T_i$ that expresses up to k-fold belief in λ-utility-proportional beliefs. Then,

$$(b_i(t_i))(c_j, t_j) = \left(\frac{1}{|C_j|} + \frac{\lambda_j}{\bar{u}_j - \underline{u}_j} (u_j(c_j, t_j) - u_j^{average}(t_j)) \right) (b_i(t_i))(t_j)$$

for all $c_j \in C_j$ and $t_j \in T_j$. Therefore,

$$(b_i(t_i))(c_j) = \sum_{t_j \in T_j(t_i) \subseteq B_j^{k-\mathbf{1}}} (b_i(t_i))(t_j) \left(\frac{1}{|C_j|} + \frac{\lambda_j}{\bar{u}_j - \underline{u}_j} (u_j(c_j, t_j) - u_j^{average}(t_j)) \right)$$

for all $c_j \in C_j$, where B_j^{k-1} denotes the set of j's types that express $k-1$ fold belief in λ-utility-proportional beliefs.

Written as vector

$$(b_i(t_i))(c_j)_{c_j \in C_j} =$$
$$\sum_{t_j \in T_j(t_i) \subseteq B_j^{k-1}} (b_i(t_i))(t_j) \left(\frac{1}{|C_j|}(1, \ldots, 1) + \frac{\lambda_j}{\bar{u}_j - \underline{u}_j}(u_j(c_j, t_j) - u_j^{average}(t_j))_{c_j \in C_j} \right).$$

Since every $t_j \in T_j(t_i)$ is in B_j^{k-1} and hence by the induction hypothesis $(b_i(t_i)) \in P_j^{k-1}$, it follows that

$$\begin{aligned} & \frac{1}{|C_j|}(1, \ldots, 1) + \frac{\lambda_j}{\bar{u}_j - \underline{u}_j}(u_j(c_j, t_j) - u_j^{average}(t_j))_{c_j \in C_j} \\ = \quad & p_i^*(b_j(t_j)) \in P_i^k. \end{aligned}$$

Since $(b_i(t_i))(c_j)_{c_j \in C_j}$ is a convex combination of elements in the convex set P_i^k, it follows that $(b_i(t_i))(c_j)_{c_j \in C_j} \in P_i^k$. By induction on k the *only if* direction of the theorem follows.

Let's now consider the if direction.

Let $p_i \in P_i^\infty$ and $p_j \in P_j^\infty$, which are unique as shown previously. Consider the epistemic model

$$\mathcal{M}^\Gamma = ((T_i, T_j), (b_i, b_j))$$

of Γ, where $T_i = \{t_i\}$ and $T_j = \{t_j\}$.

Furthermore, $b_j(t_j)$ projected on T_i puts probability 1 on t_i, while projected on C_j eqauls $p_j(p_i)$.

Note that $(b_i(t_i))(c_j|t_j) = (b_i(t_i))(c_j)$ for all $c_j \in C_j$ and $(b_j(t_j))(c_j|t_i) = (b_j(t_j))(c_j)$ for all $c_j \in C_j$, since $|T_i| = 1 = |T_j|$.

Using the fact that

$$p_i = p_i^*(p_j) \text{ and } p_j = p_j^*(p_i),$$

it then follows, by construction of b_i and b_j, that both t_i as well as t_j express λ-utility-proportional beliefs. Moveover, t_i and t_j express common belief in λ-utility-proportional beliefs too, as $T_j(t_i) = \{t_j\}$ and $T_i(t_j) = \{t_i\}$.

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Explicit Formula

$$p'_i = p^*_i(p^*_j(p'_i)) = i_m + \frac{\lambda_j}{\bar{u}_j - \underline{u}_j} C_m U_j p^*_j(p'_i)$$

Explicit Formula

$$
\begin{aligned}
p_i' = p_i^*(p_j^*(p_i')) &= i_m + \frac{\lambda_j}{\bar{u}_j - \underline{u}_j} C_m U_j p_j^*(p_i') \\
&= i_m + \frac{\lambda_j}{\bar{u}_j - \underline{u}_j} C_m U_j \left[i_n + \frac{\lambda_i}{\bar{u}_i - \underline{u}_i} C_n U_i p_i' \right]
\end{aligned}
$$

Solving for p_i' yields

$$p_i' = (I_m - \frac{\lambda_j \lambda_i}{(\bar{u}_j - \underline{u}_j)(\bar{u}_i - \underline{u}_i)} C_m U_j C_n U_i)^{-1}(i_m + \frac{\lambda_j}{\bar{u}_j - \underline{u}_j} C_m U_j i_n).$$

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Definition

We define the following function to obtain the unique beliefs under common belief in λ-*utility-proportional-beliefs* directly:

$$\beta_i(U_i, U_j) := (I_m - G_j G_i)^{-1}(i_m + G_j i_n)$$

Outline

Understanding the Players' Reasoning

Definition

We define the following function to obtain the unique beliefs under common belief in λ-*utility-proportional-beliefs* directly:

$$\beta_i(U_i, U_j) := (I_m - G_j G_i)^{-1}(i_m + G_j i_n)$$

Using the properties of a geometric series, we can rewrite the expression as:

$$\beta_i = (i_m + G_j i_n) + G_j G_i(i_m + G_j i_n) + G_j G_i \left\{ G_j G_i(i_m + G_j i_n) \right\} + \cdots$$

Initial Belief

Let's define $\beta_i^{initial} := (i_m + G_j i_n)$.

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Initial Belief

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- Firstly, the player assigns equal probability.
- Secondly, he corrects for the opponents incentives, given that the opponent assigns equal probability to all of her choices.
- G_j gives the goodness of a certain choice given a choice of the opponent.

Interaction Process

Looking back at the initial expression

$$\beta_i = \beta^{initial} + G_j G_i \beta^{initial} + G_j G_i \left\{ G_j G_i \beta^{initial} \right\} + \cdots .$$

To obtain a better understanding we rewrite β_i as follows

$$\beta_i = i_m + G_j(i_n + G_i(i_m + G_j i_n)) + \cdots .$$

Outline

The Two Systems of Reasoning

System 1

The origin of impressions and feelings that are the main sources of the explicit beliefs and deliberate choices of System 2.

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System 2

The conscious, reasoning self that has beliefs, makes choices, and decides what to think about and what to do.

[?, ?]

Relating the Systems to UPB

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- The initial belief is the basis for further reasoning steps and in many cases already a good approximation of the final belief.
- The initial belief is being adjusted step by step to form the player's final belief.
- Single steps of adaptation are fairly small (at max $1/m$) and decrease with every reasoning step.