Advanced Topics I

Incomplete Information

Christian W. Bach

EPICENTER & University of Liverpool

Agenda

- **Introduction**
- Common Belief in Rationality
- Generalized Iterated Strict Dominance
- Outlook

Games with Incomplete Information

- In games with incomplete information, players face uncertainty about their opponents' payoffs.
- Harsanyi (1967-68) pioneered the analysis of incomplete information and proposed the – now standard – **solution concept** of Bayesian equilibrium.
- Recently, the non-equilibrium **solution concept** of rationalizability has been generalized to incomplete information:
  - Δ-rationalizability – Battigalli (2003), Battigalli & Siniscalchi (2003), Battigalli et al. (2011), Dekel & Siniscalchi (2015)
  - interim rationalizability – Ely and Pęski (2006)
  - interim correlated rationalizability – Dekel et al. (2007)
- **Reasoning foundations** for the incomplete information versions of rationalizability in terms of common belief in rationality are due to Battigalli & Siniscalchi (2007) and Battigalli et al. (2011).

Payoff Uncertainty with a One-Person Perspective

- Bach & Perea (2016) follow Harsanyi's one-person perspective approach:

  > [...] *player* j *(from whose point of view we are analyzing the game)* [...] *(Harsanyi, 1967-68, p. 170)*

  > [...] *we are interested only in the decision rules that player* j *himself will follow* [...] *(Harsanyi, 1967-68, p. 175)*

- **Solution Concepts:** Bach & Perea (2016) propose generalized iterated strict dominance (GISD) as an incomplete information generalization of iterated strict dominance in terms of decision problems.

- **Reasoning Foundations:** Bach & Perea (2016) formalize common belief in rationality (CBR) with a one-person perspective approach and epistemically characterize GISD by it.

CBR and ISD in Complete Information Games

Bob

Alice		d	e	f
	a	3, 3	2, 2	1, 0
	b	2, 1	1, 2	3, 0
	c	0, 1	0, 3	0, 4

CBR and ISD in Complete Information Games

		Bob		
		d	e	f
Alice	a	$3,3$	$2,2$	$1,0$
	b	$2,1$	$1,2$	$3,0$

CBR and ISD in Complete Information Games

		Bob	
		d	e
Alice	a	$3, 3$	$2, 2$
	b	$2, 1$	$1, 2$

CBR and ISD in Complete Information Games

		Bob	
		d	e
Alice	a	$3, 3$	$2, 2$

CBR and ISD in Complete Information Games

		Bob
		d
Alice	a	$3, 3$

- Iterated strict dominance yields the strategy profile (a, d) as a **solution** to the game.
- This corresponds to **reasoning** in line with common belief in rationality.

Motivation: Extension to Incomplete Information

Alice \ Bob	d	e	f
a	3, 3	2, 2	1, 0
b	2, 2	1, 1	3, 0
c	0, 1	0, 3	0, 0

Alice \ Bob	d	e	f
a	3, 1	2, 2	1, 0
b	2, 3	1, 1	3, 0
c	0, 1	0, 1	0, 0

Alice \ Bob	d	e	f
a	1, 3	3, 2	1, 0
b	2, 2	1, 1	1, 0
c	0, 1	0, 3	0, 0

Alice \ Bob	d	e	f
a	1, 1	3, 2	1, 0
b	2, 3	1, 1	1, 0
c	0, 1	0, 1	0, 0

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Example: Going to Yet Another Party

Story:

- *Barbara* and *you* are going together to another party.
- *You* wonder what colour you should wear.
- *You* prefer *blue* (4) to *green*, *green* (3) to *red*, *red* (2) to *yellow* (1), and dislike most to wear the same colour (0) as *Barbara*.
- However, *you* drank so much at the last party, that you forgot *Barbara*'s colour preferences.
- *You* are still certain about *Barbara* also disliking most to wear the same colour (0) as you.
- Also, *you* remember that *Barbara* either prefers *red* (4) to *yellow*, *yellow* (3) to *blue*, *blue* (2) to *green* (1); **or** *blue* (4) to *yellow*, *yellow* (3) to *red*, *red* (2) to *green* (1).
- **Question:** Which colours can *you* **rationally** choose for tonight's party under common belief in rationality?

Example: Going to Yet Another Party

- *Yellow* is not optimal for *you* for any belief about *Barbara*'s choice.
- For both preferences of *Barbara* *green* is not optimal for any belief about *your* choice.
- If *you* believe *Barbara* not to choose *green* indpendent of her preferences, then *green* is better than *red* for *you*.
- If *Barbara* believes that *you* do choose neither *red* nor *yellow*, then given the first preferences only *red* is rational for *Barbara* and given the second preferences *blue* and *yellow* are rational for *Barbara*.

Static Games with Incomplete Information

Definition

A finite static game with incomplete information is a tuple $\Gamma = \big(I, (C_i)_{i \in I}, (U_i)_{i \in I}\big)$ of finite sets, such that

- I is a set of players,
- C_i is a set of player i's choices,
- U_i is a set of player i's utility functions, where $u_i : \times_{j \in I} C_j \to \mathbb{R}$ for all $u_i \in U_i$.

Possible sources for uncertainty for a player i:

- opponents' choice combinations $c_{-i} \in C_{-i}$,
- opponents' payoff combinations $u_{-i} \in U_{-i}$.

Decision Problems

- Given a game $\Gamma = \big(I, (C_i)_{i \in I}, (U_i)_{i \in I}\big)$, a decision problem for player i is a tuple

$$\Gamma_i(u_i) = (D_i, D_{-i}, u_i)$$

where $D_i \subseteq C_i$ is a subset of i's choices, $D_{-i} \subseteq C_{-i}$ is a subset of the opponents' choice combinations, and $u_i \in U_i$ is a utility function of player i.

- A decision problem is a one-person perspective model of a game-theoretic choice problem.

- A decision problem is called full, if $D_i = C_i$ and $D_{-i} = C_{-i}$, and reduced, if it is not the case that $D_i = C_i$ and $D_{-i} = C_{-i}$.

Illustration

- A game with incomplete information:

Alice \ Bob	d	e	f
a	3, 3	2, 2	1, 0
b	2, 2	1, 1	3, 0
c	0, 1	0, 3	0, 0

Alice \ Bob	d	e	f
a	3, 1	2, 2	1, 0
b	2, 3	1, 1	3, 0
c	0, 1	0, 1	0, 0

Alice \ Bob	d	e	f
a	1, 3	3, 2	1, 0
b	2, 2	1, 1	1, 0
c	0, 1	0, 3	0, 0

Alice \ Bob	d	e	f
a	1, 1	3, 2	1, 0
b	2, 3	1, 1	1, 0
c	0, 1	0, 1	0, 0

- Representation of the game in terms of decision problems:

$\Gamma_A(u_A)$	d	e	f
a	3	2	1
b	2	1	3
c	0	0	0

$\Gamma_A(u'_A)$	d	e	f
a	1	3	1
b	2	1	1
c	0	0	0

$\Gamma_B(u_B)$	a	b	c
d	3	2	1
e	2	1	3
f	0	0	0

$\Gamma_B(u'_B)$	a	b	c
d	1	3	1
e	2	1	1
f	0	0	0

- Example of a reduced decision problem for *Alice*:

$\hat{\Gamma}_A(u'_A)$	d	f
a	1	1
b	2	1

Reasoning About the Game: Belief Hierarchies

- A belief hierarchy for player i consists of
  - what i believes about his opponents' choices and payoffs (first-order belief),
  - what i believes about what his opponents believe about their opponents' choices and payoffs (second-order belief),
  - what i believes about what his opponents believe about what their opponents believe about their opponents' choices and payoffs (third-order belief),
  - etc.

- Hence, in a belief hierarchy, a player i holds a belief
  - about his opponents' choices and payoffs,
  - and about his opponents' belief hierarchies.

Epistemic Model

- Compact representation of belief hierarchies by the notion of type (Harsanyi, 1967-68)

Definition

Let $\Gamma = \big(I, (C_i)_{i\in I}, (U_i)_{i\in I}\big)$ be a game with incomplete information. An epistemic model of Γ is a tuple $\mathcal{M}^\Gamma = \big((T_i)_{i\in I}, (b_i)_{i\in I}\big)$, where for every player $i \in I$

- T_i is a finite set of types of player i,
- $b_i : T_i \to \Delta(C_{-i} \times T_{-i} \times U_{-i})$ assigns to every type $t_i \in T_i$ of player i a probability measure $b_i[t_i]$ on the set of opponents' choice type utility function combinations.

- Every type t_i in an epistemic model induces an infinite belief hierarchy.

Some Basic Epistemic Notions

- For a given type $t_i \in T_i$, and a utility function $u_i \in U_i$, the expression

$$v_i(c_i, t_i, u_i) := \sum_{c_{-i} \in C_{-i}} b_i[t_i](c_{-i}) \cdot u_i(c_i, c_{-i})$$

denotes the expected utility for choosing $c_i \in C_i$.

- A choice $c_i \in C_i$ is optimal given the type utility function pair (t_i, u_i), if

$$v_i(c_i, t_i, u_i) \geq v_i(c_i', t_i, u_i)$$

holds for all $c_i' \in C_i$.

- A type $t_i \in T_i$ is said to believe in the opponents' rationality, if $b_i[t_i]$ only assigns positive probability to triples (c_j, t_j, u_j) such that c_j is optimal given (t_j, u_j), for every opponent $j \in I \setminus \{i\}$.

Common Belief in Rationality with a One-Person Perspective Approach

Definition

Let $\Gamma = \big(I, (C_i)_{i \in I}, (U_i)_{i \in I}\big)$ be a game with incomplete information, $\mathcal{M}^\Gamma$ some epistemic model of it, $i \in I$ some player, and $t_i \in T_i$ some type of player i.

- (Induction start) Type t_i expresses 1-fold belief in rationality, if t_i believes in the opponents' rationality.
- (Induction step) For every $k > 1$, type t_i expresses k-fold belief in rationality, if t_i only assigns positive probability to opponents' types that express $(k-1)$-fold belief in rationality.

Type t_i expresses common belief in rationality, it t_i expresses k-fold belief in rationality for every $k \geq 1$.

Decision Rule: Rational Choice under Common Belief in Rationality given a Utility Function

Definition

Let $\Gamma = \big(I, (C_i)_{i \in I}, (U_i)_{i \in I}\big)$ be a game with incomplete information, $i \in I$ some player, and $u_i \in U_i$ some utility function of player i. A choice $c_i \in C_i$ of player i is rational under common belief in rationality given utility function u_i, if there exists an epistemic model $\mathcal{M}^\Gamma = \big((T_i)_{i \in I}, (b_i)_{i \in I}\big)$ of Γ with a type $t_i \in T_i$ of player i such that

- c_i is optimal given (t_i, u_i),
- t_i expresses common belief in rationality.

Illustration

$\Gamma_A(u_A)$

	d	e	f
a	3	2	1
b	2	1	3
c	0	0	0

$\Gamma_A(u'_A)$

	d	e	f
a	1	3	1
b	2	1	1
c	0	0	0

$\Gamma_B(u_B)$

	a	b	c
d	3	2	1
e	2	1	3
f	0	0	0

$\Gamma_B(u'_B)$

	a	b	c
d	1	3	1
e	2	1	1
f	0	0	0

- Consider the following epistemic model of the game:
  - $T_{Alice} = \{t_A, t'_A\}$ and $T_{Bob} = \{t_B, t'_B\}$,
  - $b_{Alice}[t_A] = (d, t_B, u_B)$ and $b_{Alice}[t'_A] = (e, t'_B, u'_B)$,
  - $b_{Bob}[t_B] = (a, t_A, u_A)$ and $b_{Bob}[t'_B] = \frac{1}{2}(b, t_A, u'_A) + \frac{1}{2}(c, t'_A, u_A)$.
- Types t_A and t_B believe in the opponents' rationality.
- Types t_A and t_B express common belief in rationality.

Example: Going to Yet Another Party

$\Gamma_y^0(u_y)$

	blue	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2
yellow	1	1	1	0

$\Gamma_B^0(u_B)$

	blue	*green*	*red*	*yellow*
blue	0	2	2	2
green	1	0	1	1
red	4	4	0	4
yellow	3	3	3	0

$\Gamma_B^0(u_B')$

	blue	*green*	*red*	*yellow*
blue	0	4	4	4
green	1	0	1	1
red	2	2	0	2
yellow	3	3	3	0

- Consider the following epistemic model of the game:
  - $T_{you} = \{t_y^b, t_y^g\}$ and $T_{Barbara} = \{t_B^b, t_B^r\}$,
  - $b_{you}[t_y^b] = (red, t_B^r, u_B)$ and $b_{you}[t_y^g] = (blue, t_B^b, u_B')$,
  - $b_{Barbara}[t_B^b] = (green, t_y^g, u_y)$ and $b_{Barbara}[t_B^r] = (blue, t_y^b, u_y)$.
- All types believe in the opponents' rationality, and thus also express common belief in rationality.
- Consequently, *you* can rationally choose *blue* as well as *green* under common belief in rationality given utility function u_y.

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Definition of GISD (Part I of II)

- Let $\Gamma = \big(I, (C_i)_{i \in I}, (U_i)_{i \in I}\big)$ be a game with incomplete information.

- For every player $i \in I$ and for every utility function $u_i \in U_i$, define the initial decision problem

$$\Gamma_i^0(u_i) := (C_i, C_{-i}, u_i).$$

- Round 1: For every player $i \in I$ and for every utility function $u_i \in U_i$, define the 1-fold reduced decision problem

$$\Gamma_i^1(u_i) := (C_i^1(u_i), C_{-i}^1(u_i), u_i)$$

  where

  - $C_{-i}^1(u_i) = C_{-i}$,
  - $C_i^1(u_i)$ is obtained from C_i by eliminating all choices that are strictly dominated in the decision problem $(C_i, C_{-i}^1(u_i), u_i)$.

Definition of GISD (Part II of II)

- Round 2: For every player $i \in I$ and for every utility function $u_i \in U_i$, define the 2-fold reduced decision problem

$$\Gamma_i^2(u_i) := (C_i^2(u_i), C_{-i}^2(u_i), u_i)$$

where

  - $C_{-i}^2(u_i)$ is obtained from $C_{-i}^1(u_i)$ by eliminating, for every opponent $j \in I \setminus \{i\}$, all choices that are not in $C_j^1(u_j)$ for any utility function $u_j \in U_j$,
  - $C_i^2(u_i)$ is obtained from $C_i^1(u_i)$ by eliminating all choices that are strictly dominated in the decision problem $(C_i^1(u_i), C_{-i}^2(u_i), u_i)$.

- etc.

- For every player $i \in I$, the output $GISD_i \subseteq C_i \times U_i$ only contains choice utility function pairs (c_i, u_i) s.t. $c_i \in C_i^k(u_i)$ for all $k \geq 0$.

Characterization

Theorem

Let $\Gamma = \big(I, (C_i)_{i \in I}, (U_i)_{i \in I}\big)$ *be a game with incomplete information,* $i \in I$ *some player,* $c_i \in C_i$ *some choice, of player* i*, and* $u_i \in U_i$ *some utility function of player* i*. The choice* c_i *is rational under common belief in rationality given* u_i*, if and only if,* $(c_i, u_i) \in GISD_i$*.*

Illustration

Initial decision problems:

$\Gamma_A^0(u_A)$

	d	e	f
a	3	2	1
b	2	1	3
c	0	0	0

$\Gamma_A^0(u_A')$

	d	e	f
a	1	3	1
b	2	1	1
c	0	0	0

$\Gamma_B^0(u_B)$

	a	b	c
d	3	2	1
e	2	1	3
f	0	0	0

$\Gamma_B^0(u_B')$

	a	b	c
d	1	3	1
e	2	1	1
f	0	0	0

Round 1:

- For *Alice*, choice c is strictly dominated in the initial decision problems $\Gamma_A^0(u_A)$ and $\Gamma_A^0(u_A')$.
- Eliminate choice c from $\Gamma_A^0(u_A)$ and from $\Gamma_A^0(u_A')$.
- Similarly, eliminate choice f from *Bob*'s initial decision problems $\Gamma_B^0(u_B)$ and $\Gamma_B^0(u_B')$.

Illustration

1-fold reduced decision problems:

$\Gamma_A^1(u_A)$

	d	e	f
a	3	2	1
b	2	1	3

$\Gamma_A^1(u'_A)$

	d	e	f
a	1	3	1
b	2	1	1

$\Gamma_B^1(u_B)$

	a	b	c
d	3	2	1
e	2	1	3

$\Gamma_B^1(u'_B)$

	a	b	c
d	1	3	1
e	2	1	1

Round 2:

- For *Bob*, choice f is not in any of his decision problems $\Gamma_B^1(u_B)$ and $\Gamma_B^1(u'_B)$.
- Eliminate choice f from $\Gamma_A^1(u_A)$ and from $\Gamma_A^1(u'_A)$.
- Similarly, eliminate choice c from *Bob*'s 1-fold reduced decision problems $\Gamma_B^1(u_B)$ and $\Gamma_B^1(u'_B)$.

Illustration

$(C_A^1(u_A), C_B^2(u_A), u_A)$

	d	e
a	3	2
b	2	1

$(C_A^1(u'_A), C_B^2(u'_A), u'_A)$

	d	e
a	1	3
b	2	1

$(C_B^1(u_B), C_A^2(u_B), u_B)$

	a	b
d	3	2
e	2	1

$(C_B^1(u'_B), C_A^2(u'_B), u'_B)$

	a	b
d	1	3
e	2	1

Round 2:

- Then, choice b becomes strictly dominated for *Alice* in the decision problem $\big(C_A^1(u_A), C_B^2(u_A), u_A\big)$.
- Eliminate choice b from the decision problem $\big(C_A^1(u_A), C_B^2(u_A), u_A\big)$.
- Similarly, eliminate choice e from *Bob*'s decision problem $\big(C_B^1(u_B), C_A^2(u_B), u_B\big)$.

Illustration

2-fold reduced decision problems:

$\Gamma^2_A(u_A)$

	d	e
a	3	2

$\Gamma^2_A(u'_A)$

	d	e
a	1	3
b	2	1

$\Gamma^2_B(u_B)$

	a	b
d	3	2

$\Gamma^2_B(u'_B)$

	a	b
d	1	3
e	2	1

Round 3:

- The algorithm stops.
- Output:

$$GISD_{Alice} = \{(a,u_A),(a,u'_A),(b,u'_A)\}$$

and

$$GISD_{Bob} = \{(d,u_B),(d,u'_B),(e,u'_B)\}.$$

Example: Going to Yet Another Party

Initial decision problems:

$\Gamma_y^0(u_y)$

	blue	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2
yellow	1	1	1	0

$\Gamma_B^0(u_B)$

	blue	*green*	*red*	*yellow*
blue	0	2	2	2
green	1	0	1	1
red	4	4	0	4
yellow	3	3	3	0

$\Gamma_B^0(u_B')$

	blue	*green*	*red*	*yellow*
blue	0	4	4	4
green	1	0	1	1
red	2	2	0	2
yellow	3	3	3	0

Round 1:

- For *you*, choice $yellow$ is strictly dominated in the initial decision problem $\Gamma_y^0(u_y)$ by $\frac{1}{2}blue + \frac{1}{2}green$.
- Eliminate choice $yellow$ from $\Gamma_y^0(u_y)$.
- For *Bob*, choice $green$ is strictly dominated by $\frac{1}{2}red + \frac{1}{2}yellow$ in $\Gamma_B^0(u_B)$ and by $\frac{1}{2}blue + \frac{1}{2}yellow$ in $\Gamma_B^0(u_B')$.
- Eliminate choice $green$ from $\Gamma_B^0(u_B)$ and from $\Gamma_B^0(u_B')$.

Example: Going to Yet Another Party

1-fold reduced decision problems:

$\Gamma_y^1(u_y)$

	blue	green	red	yellow
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2

$\Gamma_B^1(u_B)$

	blue	green	red	yellow
blue	0	2	2	2
red	4	4	0	4
yellow	3	3	3	0

$\Gamma_B^1(u_B')$

	blue	green	red	yellow
blue	0	4	4	4
red	2	2	0	2
yellow	3	3	3	0

Round 2:

- For *Barbara*, choice *green* is not in any of her decision problems $\Gamma_B^1(u_B)$ and $\Gamma_B^1(u_B')$.
- Eliminate choice *green* from $\Gamma_y^1(u_y)$.
- For *you*, choice *yellow* is not in your (only) decision problem $\Gamma_y^1(u_y)$.
- Eliminate choice *yellow* from $\Gamma_B^1(u_B)$ and from $\Gamma_B^1(u_B')$.

Example: Going to Yet Another Party

$(C_y^1(u_y), C_B^2(u_y), u_y)$

	blue	*red*	*yellow*
blue	0	4	4
green	3	3	3
red	2	0	2

$(C_B^1(u_B), C_y^2(u_B), u_B)$

	blue	*green*	*red*
blue	0	2	2
red	4	4	0
yellow	3	3	3

$(C_B^1(u'_B), C_y^2(u'_B), u'_B)$

	blue	*green*	*red*
blue	0	4	4
red	2	2	0
yellow	3	3	3

Round 2:

- Choice *red* becomes strictly dominated by *green* for *you* in the decision problem $(C_y^1(u_y), C_B^2(u_y), u_y)$: eliminate choice *red* from $(C_y^1(u_y), C_B^2(u_y), u_y)$.
- Choice *blue* becomes strictly dominated by *yellow* for *Barbara* in the decision problem $(C_B^1(u_B), C_y^2(u_B), u_B)$: eliminate choice *blue* from $(C_B^1(u_B), C_y^2(u_B), u_B)$.
- Choice *red* becomes strictly dominated by *yellow* for *Barbara* in the decision problem $(C_B^1(u'_B), C_y^2(u'_B), u'_B)$: eliminate choice *red* from $(C_B^1(u'_B), C_y^2(u'_B), u'_B)$.

Example: Going to Yet Another Party

2-fold reduced decision problems:

$\Gamma_y^2(u_y)$

	blue	*red*	*yellow*
blue	0	4	4
green	3	3	3

$\Gamma_B^2(u_B)$

	blue	*green*	*red*
red	4	4	0
yellow	3	3	3

$\Gamma_B^2(u_B')$

	blue	*green*	*red*
blue	0	4	4
yellow	3	3	3

Round 3:

- For *Barbara*, *blue*, *red* and *yellow* are each in some round 2 decision problem of *Barbara*.
- For *you*, choice red is not in your (only) round 2 decision problem $\Gamma_y^2(u_y)$.
- Eliminate choice red from $\Gamma_B^2(u_B)$ and from $\Gamma_B^2(u_B')$.

Example: Going to Yet Another Party

$\Gamma_y^2(u_y)$

	blue	*red*	*yellow*
blue	0	4	4
green	3	3	3

$(C_B^2(u_B), C_A^3(u_B), u_B)$

	blue	*green*
red	4	4
yellow	3	3

$(C_B^2(u'_B), C_A^3(u'_B), u'_B)$

	blue	*green*
blue	0	4
yellow	3	3

Round 2:

- Choice $yellow$ becomes strictly dominated by red for *Barbara* in the decision problem $\left(C_B^2(u_B), C_y^3(u_B), u_B\right)$: eliminate choice $yellow$ from $\left(C_B^2(u_B), C_y^3(u_B), u_B\right)$.

Example: Going to Yet Another Party

3-fold reduced decision problems:

$\Gamma^3_y(u_y)$

	blue	*red*	*yellow*
blue	0	4	4
green	3	3	3

$\Gamma^3_B(u_B)$

	blue	*green*
red	4	4

$\Gamma^3_B(u'_B)$

	blue	*green*
blue	0	4
yellow	3	3

Round 4:

- The algorithm stops.
- Output:

$$GISD_{you} = \{(blue, u_y), (green, u_y)\}$$

and

$$GISD_{Barbara} = \{(red, u_B), (blue, u'_B), (yellow, u'_B)\}.$$

Agenda

- Introduction
- Generalized Iterated Strict Dominance
- Reasoning Foundations
- **Outlook**

Future Research

- Extension of Perea's (2014) common belief in future rationality to incomplete information dynamic games.

- Cautious reasoning with incomplete information: extension of Brandenburger et al.'s (2008) common assumption of rationality to incomplete information.

- Numerous applications such as Industrial Organization, Mechanism Design, Business Strategy, etc.

The Bigger Picture (Bach & Perea, 2017a)

- Solution concepts for static games in relation to **informational assumptions** and *epistemic conditions*:

	Complete Information	**Incomplete Information**
CBR	Rat & ISD	Δ-Rat & GISD
	$\not\Downarrow$ / $\Uparrow$	$\not\Downarrow$ / $\Uparrow$
CBR & Common Prior Assumption	Correlated Equilibrium	Bayesian Equilibrium
	$\not\Downarrow$ / $\Uparrow$	$\not\Downarrow$ / $\Uparrow$
CBR & Simple Belief Hierarchy	Nash Equilibrium	Generalized Nash Equilibrium

- In fact, the epistemic characterization of Generalized Nash Equilibrium can be weakened to conditions that do not even imply CBR (Bach & Perea, 2017b).

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