Multidimensional Possible world models

Richard Bradley

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- (*Marginalisation*):

$$p_A(w_j) = \sum_{w_i \in W} p(\langle w_i, w_j \rangle)$$
$$p_w(w_i) = \sum_{w_j \in W_A} p(\langle w_i, w_j \rangle)$$

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- Marginalisation implies that:

$$\begin{aligned} V_w(X) &= j.V(X) \\ V_A(X) &= k_A.V(A \rightarrow X) \end{aligned}$$

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- From the assumption of Restricted Actualism and Prospect Actualism it follows that:
    1. Additivity: $V(B_A, C_{A'}) = V(B_A) + V(C_{A'})$
    2. Independence: $V((A \mapsto B)|(\neg A \mapsto C)) = V(A \mapsto B)$

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- *SEU Hypothesis*: Let $\{\alpha_i\}$ be an n-fold partition. Then:

$$V((\alpha_1 \mapsto \beta_1)(\alpha_2 \mapsto \beta_2)...(\alpha_n \mapsto \beta_n)) = \sum_{i=1}^{n} V(\alpha_i\beta_i).P(\alpha_i)$$

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Theorem

Assume Desirabilism. Then the conjunction of Bradley's Thesis and the Independence condition is equivalent to the SEU hypothesis.

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- Joyce: CU is desirability on the supposition of action's performance

$$\begin{aligned} V_A(X) &= \sum_{w_i \in W} v(w_i).P_A(w_i|X) \\ CU(A) &= V_A(A) \end{aligned}$$

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- An alternative proposal: Maximise expected desirability gain, i.e.

$$V^{*A}(X) = \sum_{w_i \in X} v(w_i).(\frac{p^*_A(w_i) - p(w_i)}{P^*_A(X)})$$