

ECON322 Game Theory

Half II Problem Set 2



Question 1

Suppose you wish to open a supermarket in an area with three little villages: Colmont, Winthagen and Ransdaal. Colmont has 300 inhabitants, Winthagen has 200 inhabitants, and Ransdaal has 400 inhabitants. Every inhabitant is a potential customer. There are four possible locations for the supermarket, which we call a ; b ; c and d . Figure 1 provides a map of the area with the scale 1 : 50.000. It shows how the villages and the possible locations are situated. However, there is a competitor who also wishes to open a supermarket in the same area. Once you and your competitor have chosen a location, every inhabitant will always visit the supermarket that is closest to his village. If you happen to choose the same location, you will share the market equally with him.

- (a) Use the algorithm of iterated strict dominance to determine the locations you and your competitor can rationally choose under common belief in rationality.
- (b) Construct an epistemic model such that, for each of the locations c_i found in (a) there is a type $t_i^{c_i}$ such that:
 - location c_i is optimal for $t_i^{c_i}$,
 - and type $t_i^{c_i}$ expresses common belief in rationality.
- (c) For each of my types in the epistemic model constructed in (b), describe the induced belief hierarchy.

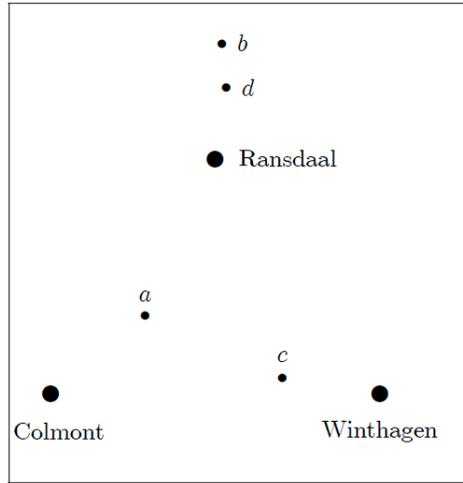


Fig. 1. Map of the area

Question 2

Barbara, Chris and you are sitting in a bar, having a drink before the movie starts. You have brought a pack of playing cards with you, and tell your friends about a new card game you invented last night. The rules are easy: There are three piles of cards on the table with their faces down. One pile contains the 2, 5, 8 and jack of hearts, another pile contains the 3, 6, 9 and queen of hearts, and the last pile contains the 4, 7, 10 and king of hearts, and everybody knows this. The jack is worth 11 points, the queen is worth 12 points and the king 13 points. Each of the three players receives one of these decks, and everybody knows the decks that are given to the other two players. Then, all players simultaneously choose one card from their deck, and put it on the table. The player putting the card whose value is the middle value on the table wins the game. Every losing player pays the value of his own card in euros to the winning player. Suppose that when you start playing the game, you hold the deck with the 3, 6, 9 and the queen.

- (a) Use the algorithm of iterated strict dominance to determine the cards you, Barbara, and Chris can rationally choose under common belief in rationality.
- (b) Construct an epistemic model such that, for each of the cards c_i found in
- (a) there is a type $t_i^{c_i}$ such that:
- card c_i is optimal for $t_i^{c_i}$,

- and type $t_i^{c_i}$ expresses common belief in rationality.

Question 3

This evening Barbara has invited you for dinner. You promised her to bring something to drink, and as usual you either bring some bottles of beer, or a bottle of white wine, or a bottle of red wine. Barbara's favourite dishes are salmon, souvlaki, and nasi goreng. Of course, you want to bring a drink that combines well with the dish that Barbara prepares. Both – you and Barbara – agree that salmon combines reasonably well with beer, combines badly with red wine, but provides an excellent combination with white wine. You also agree that souvlaki combines reasonably well with beer, combines badly with white wine, but provides an excellent combination with red wine. Finally, you agree that nasi goreng provides a reasonable combination with white wine and red wine. However, you find that nasi goreng combines excellently with beer, whereas according to Barbara nasi goreng only provides a reasonable combination with beer. Suppose that a bad combination gives a utility of 0, that a reasonable combination yields a utility of 1, and that an excellent combination gives a utility of 3.

- (a) Formulate the story as a game between you and Barbara.
- (b) Use the algorithm of iterated strict dominance to determine the drinks you and Barbara can rationally choose under common belief in rationality.
- (c) Construct an epistemic model such that, for each of the drinks c_i found in (b) there is a type $t_i^{c_i}$ such that:
 - drink c_i is optimal for $t_i^{c_i}$,
 - and type $t_i^{c_i}$ expresses common belief in rationality.