

ECON322 Game Theory

Half II Problem Set 1



Question 1

Suppose you wish to open a supermarket in an area with three little villages: Colmont, Winthagen and Ransdaal. Colmont has 300 inhabitants, Winthagen has 200 inhabitants, and Ransdaal has 400 inhabitants. Every inhabitant is a potential customer. There are four possible locations for the supermarket, which we call a ; b ; c and d . Figure 1 provides a map of the area with the scale 1 : 50.000. It shows how the villages and the possible locations are situated. However, there is a competitor who also wishes to open a supermarket in the same area. Once you and your competitor have chosen a location, every inhabitant will always visit the supermarket that is closest to his village. If you happen to choose the same location, you will share the market equally with him.

- (a) Formulate the story as a game between you and a competitor.

- (b) Which locations are rational for you, and which are not? For every rational location, find a belief about the competitor's choice for which this location is optimal. For every irrational location, explain why there can be no belief for which this location is optimal.

- (c) Determine those locations you can rationally choose while believing that the competitor chooses rationally as well.

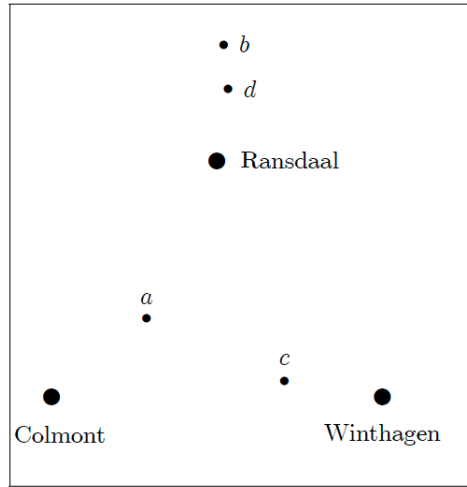


Fig. 1. Map of the area

Question 2

In your village there are two cinemas that attract visitors from all over the neighborhood. You are the manager of one of these cinemas, Cinemax. Clearly, your number of visitors will not only depend on the price you choose for a ticket, but also on the price chosen by your competitor. More precisely, if you choose a price P_1 (in EUR), and your competitor chooses a price P_2 , then your total number of customers per week will approximately be

$$600 - 60 \cdot P_1 + 30 \cdot P_2.$$

So, if you increase your price by 1 euro, you will lose 60 customers per week, and if your rival increases its price by 1 euro, you will have 30 extra customers per week. The city council has decided that the maximum price for a ticket is 10 euros. Your goal is to maximize your weekly profit.

- (a) Suppose you believe that the rival will choose a price of 4 euros. Describe your profit as a function of P_1 , and draw this function as a graph. What is your optimal price in this case? Do the same for the cases where you believe that the rival will choose a price of 0 euros, 7 euros, 9 euros and 10 euros. Draw all the functions on the same figure!

- (b) Now, suppose that you believe that the rival will choose a price of P_2 , where P_2 can be any price between 0 and 10 GBP. Describe your profit as a function of P_1 and P_2 . What is your optimal choice in this case, and how does it depend on P_2 ?
- (c) What are the rational prices that you can choose? For every rational price, find a belief about the rival's choice for which this price is optimal. For every irrational price, find another price that strictly dominates it.

Question 3

Barbara, Chris and you are sitting in a bar, having a drink before the movie starts. You have brought a pack of playing cards with you, and tell your friends about a new card game you invented last night. The rules are easy: There are three piles of cards on the table with their faces down. One pile contains the 2, 5, 8 and jack of hearts, another pile contains the 3, 6, 9 and queen of hearts, and the last pile contains the 4, 7, 10 and king of hearts, and everybody knows this. The jack is worth 11 points, the queen is worth 12 points and the king 13 points. Each of the three players receives one of these decks, and everybody knows the decks that are given to the other two players. Then, all players simultaneously choose one card from their deck, and put it on the table. The player putting the card whose value is the middle value on the table wins the game. Every losing player pays the value of his own card in euros to the winning player. Suppose that when you start playing the game, you hold the deck with the 3, 6, 9 and the queen.

- (a) Which cards are rational for you? For every rational card, find a belief about your friends' choices for which this card is optimal. For every irrational card, find another card, or a randomization over cards, that strictly dominates it.
- (b) Which cards are rational for your two friends? For every rational card, find a belief for which this card is optimal. For every irrational card, find another card, or a randomization over cards, that strictly dominates it.
- (c) Determine those cards you can rationally put on the table if you believe that your friends will play rationally too.