ECON322 Game Theory (Half II)

Topic 2: Common Belief in Rationality

Christian W. Bach

University of Liverpool & EPICENTER Maastricht

Interactive Reasoning

- Since the outcome in a game for a player does not only depend on his own decision, but also on what his opponents are doing, it is crucial to model his belief about his opponents' choices.
- Due to this intuition the notion of conjecture was presented in Topic 1.
- However, a full account of interactive thinking actually require mores:
  - *what a player thinks his opponents are conjecturing,*
  - *what he thinks his opponents are thinking their respective opponents are conjecturing,*
  - *etc.*
- Accordingly, **interactive reasoning** encompasses an (infinite) sequence of iterated beliefs.

Belief hierarchies

- More precisely, in **Epistemic Game Theory**, every player *i* is assumed to entertain a belief hierarchy:
  - *a belief of i about his opponents' choice-combinations,*
    (conjecture; also called first-order belief)
  - *a belief of i about his opponents' beliefs about their respective opponents', choice-combinations,*
    (second-order belief)
  - *a belief of i about his opponents' beliefs about their respective opponents' beliefs about their respective opponents' choice-combinations,*
    (third-order belief)
  - *etc.*

Thinking about Rationality Interactively

- A choice is **rational**, if it is optimal for *some* conjecture (cf. Topic 1).
- The idea of **rationality** can be infused into interactive thinking.
- More formally speaking, belief in **rationality** can be iterated throughout the entire belief hierarchy of a player.
- Actually, the **epistemic condition** of **common belief in rationality** does exactly so:
  - *player i believes his opponents to choose **rationally**,*
  - *player i believes his respective opponents to believe their respective opponents to choose **rationally**,*
  - *player i believes his respective opponents to believe their respective opponents to believe their respective opponents to choose **rationally**,*
  - *etc.*

Example I: Going to a Party

Story:

- *Alice* and *Bob* are going together to a party tonight.
- *Alice* asks herself what colour she should wear.
- *Alice* prefers *blue* to *green*, *green* to *red*, and *red* to *yellow*.
- However, *Alice* dislikes most to wear the same colour as *Bob*.
- Let the utilities be given as follows:
  - *blue*: *Alice*: 4 and *Bob*: 2
  - *green*: *Alice*: 3 and *Bob*: 1
  - *red*: *Alice*: 2 and *Bob*: 4
  - *yellow*: *Alice*: 1 and *Bob*: 3
  - same colour: *Alice*: 0 and *Bob*: 0
- Question: Which colours can *Alice* **rationally** choose for tonight's party under **common belief in rationality**?

Example I: Going to a Party

- Rational choices for *Alice*: blue, green, and red.
- Rational choices for *Bob*: red, yellow, and blue.
  - Red is optimal for *Bob*, if he believes *Alice* to choose any other colour than red.
  - Yellow is optimal for *Bob*, if he believes *Alice* to choose red.
  - Blue is optimal for *Bob*, if he believes with probability 0.6 that *Alice* chooses red and with probability 0.4 that *Alice* chooses yellow.
  - Green is never optimal: red is better for all beliefs with probability of less than 0.5 for *Alice* choosing red and yellow is better for all beliefs with probability of at least 0.5 for *Alice* choosing red.
- If *Alice* believes in *Bob*'s rationality, then she assigns probability 0 to *Bob*'s choice green.
- Thus, restrict *Alice*'s belief about *Bob*'s choice to red, yellow, and blue.
  - blue is optimal, if *Alice* believes *Bob* to choose red.
  - green is optimal, if *Alice* believes *Bob* to choose blue.
  - green yields higher expected utility than red, if *Alice* believes *Bob* to choose from $\{red, yellow, blue\}$.
- Consequently, *Alice* can only rationally choose blue and green, if she believes in *Bob*'s rationality.

Example I: Going to a Party

- Rational choices for *Alice* if she believes in *Bob*'s rationality: blue, and green.
- Rational choices for *Bob* if he believes in *Alice*'s rationality: red, and yellow.
  - red is optimal, if *Bob* believes *Alice* to choose blue.
  - yellow is optimal, if *Bob* believes *Alice* to choose red.
  - yellow yields higher expected utility than blue, if *Bob* believes *Alice* to choose from $\{blue, green, red\}$.
- Can *Alice* **rationally** choose blue and green under **common belief in rationality**?

Example I: Going to a Party

- Note that blue is optimal for *Alice*, if she believes *Bob* to choose red, **and** that red is optimal for *Bob*, if he believes *Alice* to choose blue.
- Consider the following belief hierarchy h_{Alice} for *Alice*.
  - *Alice* believes *Bob* to choose red.
  - *Alice* believes *Bob* to believe her to choose blue.
  - *Alice* believes *Bob* to believe her to believe that he chooses red.
  - *Alice* believes *Bob* to believe her to believe him to believe that she chooses blue.
  - etc.
- Thus, *Alice* believes *Bob* to choose rationally, and believes *Bob* to believe her to choose rationally, etc.
- In other words, h_{Alice} does not contain any belief of any order in which the rationality neither of *Alice* nor of *Bob* is questioned.
- Consequently, h_{Alice} satisfies common belief in rationality, and blue is optimal for her given the first-order belief of h_{Alice}.

Example I: Going to a Party

- What about *Alice*'s second most preferred colour green?
- If *Alice* believes in *Bob*'s rationality, and believes that he believes in her rationality, then she assigns probability 0 to *Bob*'s choices blue and green.
- However, blue then yields higher expected utility than green for *Alice*, if she believes *Bob* to choose from $\{red, yellow\}$.
- In particular, *Alice* can hence not **rationally** choose green – but only blue– under **common belief in rationality**.
- Analogously, it can be shown that *Bob* can only **rationally** choose red under **common belief in rationality**.

Example II *(For Self-Study)*: Where to Locate My Pub?

Story:

- *Alice* and *Bob* both want to open a new pub on Bold Street.
- Bold Street contains 600 houses, equally spaced.
- One person per house is assumed to visit the closest pub.
- There are seven possible locations for pubs: a, b, c, d, e, f, g.
- If *Alice* and *Bob* choose the same location, then each gets 300 clients.
- **Question:** Which locations can *Alice* **rationally** choose under common belief in rationality?

Example II *(For Self-Study)*: Where to Locate My Pub?

- Rational strategies for *Alice*: *b*, *c*, *d*, *e*, and *f*
- Rational strategies for *Bob* (by symmetry of the game): *b*, *c*, *d*, *e*, and *f*
- If *Alice* believes in *Bob*'s rationality, then she assigns zero probability to *Bob*'s strategies *a* and *g*.
- So, restrict *Alice*'s belief about *Bob*'s strategy to *b*, *c*, *d*, *e*, and *f*.
    - *c* is optimal, if *Alice* believes *Bob* to choose *b*.
    - *d* is optimal, if *Alice* believes *Bob* to choose *d*.
    - *e* is optimal, if *Alice* believes *Bob* to choose *f*.
    - *b* is strictly dominated by *c*, if *Alice* believes *Bob* to choose from $\{b, c, d, e, f\}$.
    - *f* is strictly dominated by *e*, if *Alice* believes *Bob* to choose from $\{b, c, d, e, f\}$.
- Consequently, *Alice* can only rationally choose *c*, *d*, and *e*, if she believes in *Bob*'s rationality.

Example II *(For Self-Study)*: Where to Locate My Pub?

- Rational strategies for *Alice*, if she believes in *Bob*'s rationality:
  c, *d*, and *e*.

- Rational strategies for *Bob*, if he believes in *Alice*'s rationality (by symmetry of the game): *c*, *d*, and *e*

- If *Alice* believes in *Bob*'s rationality and believes that he believes in her rationality, then she assigns zero probability to *Bob*'s strategies *a*, *b*, *f* and *g*.

- Thus, restrict *Alice*'s belief about *Bob*'s strategy to *c*, *d*, and *e*.
  - *d* is optimal, if *Alice* believes *Bob* to choose *d*.
  - *c* is strictly dominated by *d*, if *Alice* believes *Bob* to choose from $\{c, d, e\}$.
  - *e* is strictly dominated by *d*, if *Alice* believes *Bob* to choose from $\{c, d, e\}$.

- Consequently, *Alice* can only rationally choose *d*, if she believes in *Bob*'s rationality and believes *Bob* to believe in her rationality.

Example II *(For Self-Study)*: Where to Locate My Pub?

- Can *Alice* **rationally** choose *d* under common belief in rationality?

- Note that *d* is optimal for *Alice*, if she believes *Bob* to choose *d*, **and** that *d* is optimal for *Bob*, if he believes *Alice* to choose *d*.

- Consider the following belief hierarchy h_{Alice} for *Alice*.
    - *Alice* believes *Bob* to choose *d*.
    - *Alice* believes *Bob* to believe her to choose *d*.
    - *Alice* believes *Bob* to believe her to believe him to choose *d*.
    - etc.

- Thus *Alice* believes *Bob* to choose rationally, and believes *Bob* to believe her to choose rationally, etc.

- In other words, h_{Alice} does not contain any belief of any order in which the rationality neither of *Alice* nor of *Bob* is questioned.

- Consequently, h_{Alice} satisfies common belief in rationality, and *d* is optimal for her given the first-order belief of h_{Alice}.

Agenda

- Epistemic Model
- Common Belief in Rationality
- Iterated Strict Dominance
- Characterization of Common Belief in Rationality

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- **Epistemic Model**
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Rewriting Belief Hierarchies

- A belief hierarchy involves infinitely many layers.
  - FIRST-ORDER BELIEF: *i's belief about his opponents' choices.*
  - SECOND-ORDER BELIEF: *i's belief about his opponents' beliefs about their resepctive opponents' choices.*
  - THIRD-ORDER BELIEF: *i's belief about his opponents' beliefs about their respective opponents' beliefs about their respective opponents' choices.*
  - FOURTH-ORDER BELIEF: *i's belief about his opponents' beliefs about their respective opponents' beliefs about their respective opponents' beliefs about their respective opponents' choices.*
  - *etc.*
- The above doxastic sequence can be rewritten as follows:
  - FIRST-ORDER BELIEF: *i's belief about his opponents' choices.*
  - SECOND-ORDER BELIEF: *i's belief about his opponents'* FIRST-ORDER BELIEFS.
  - THIRD-ORDER BELIEF: *i's belief about his opponents'* SECOND-ORDER BELIEFS.
  - FOURTH-ORDER BELIEF: *i's belief about his opponents'* THIRD-ORDER BELIEFS.
  - *etc.*
- In a way, a belief hierarchy thus consists of a **first-order belief** and a **belief** about the opponents´ **belief hierarchies**.

Finite Representation of Belief Hierarchies

- This is a crucial insight that actually enables a **compact representation** of belief hierarchies.
- The infinite doxastic sequences constituting a belief hierarchy is labelled by the abstract notion of **type**.
- A **type** induces a belief about the opponents' choice-type combinations.
- Every layer of the belief hierarchy that corresponds to the **type** can then be inferred.
- Types can thus be viewed as **implicit** belief hierarchies.

Epistemic Model

- **Types** and their beliefs are modelled in an additional mathematical structure called

  epistemic model

  that complements the game structure given by Γ.

Definition 1

Let Γ_N be a normal form. An epistemic model $\mathcal{M}^{\Gamma} = (T_i, b_i)_{i \in I}$ of Γ provides for every player $i \in I$,

- a finite set T_i of types,
- and for every type $t_i \in T_i$ a probability measure

$$b_i(t_i) \in \Delta\left((C_j \times T_j)_{j \in I \setminus \{i\}}\right)$$

on the opponents' choice-type combinations.

- Note that the probability measure b_i – the belief function of player i – provides for every **type** $t_i \in T_i$ a first-order belief as well as a belief about the opponents' types, i.e. belief hierarchies.

Illustration: An Epistemic Model for Example I

- Type Sets:

$$T_{Alice} = \{t^1_{Alice}, t^2_{Alice}, t^3_{Alice}\}$$
$$T_{Bob} = \{t^1_{Bob}, t^2_{Bob}, t^3_{Bob}, t^4_{Bob}\}$$

- Beliefs for *Alice*:

$$b_{Alice}(t^1_{Alice}) = (green, t^1_{Bob})$$
$$b_{Alice}(t^2_{Alice}) = (blue, t^2_{Bob})$$
$$b_{Alice}(t^3_{Alice}) = 0.6 \cdot (blue, t^3_{Bob}) + 0.4 \cdot (green, t^4_{Bob})$$

- Beliefs for *Bob*:

$$b_{Bob}(t^1_{Bob}) = (blue, t^1_{Alice})$$
$$b_{Bob}(t^2_{Bob}) = (green, t^2_{Alice})$$
$$b_{Bob}(t^3_{Bob}) = (red, t^3_{Alice})$$
$$b_{Bob}(t^4_{Bob}) = (yellow, t^1_{Alice})$$

Illustration: An Epistemic Model for Example I

- Type Sets:
  $T_{Alice} = \{t^1_{Alice}, t^2_{Alice}, t^3_{Alice}\}$
  $T_{Bob} = \{t^1_{Bob}, t^2_{Bob}, t^3_{Bob}, t^4_{Bob}\}$
- Beliefs for *Alice*:
  $b_{Alice}(t^1_{Alice}) = (green, t^1_{Bob})$
  $b_{Alice}(t^2_{Alice}) = (blue, t^2_{Bob})$
  $b_{Alice}(t^3_{Alice}) = 0.6 \cdot (blue, t^3_{Bob}) + 0.4 \cdot (green, t^4_{Bob})$
- Beliefs for *Bob*:
  $b_{Bob}(t^1_{Bob}) = (blue, t^1_{Alice})$
  $b_{Bob}(t^2_{Bob}) = (green, t^2_{Alice})$
  $b_{Bob}(t^3_{Bob}) = (red, t^3_{Alice})$
  $b_{Bob}(t^4_{Bob}) = (yellow, t^1_{Alice})$

Type t^3_{Alice} induces the following belief hierarchy:

- *Alice* believes with probability-0.6 *Bob* to wear blue and with probability-0.4 *Bob* to wear green. (**first-order belief**)
- *Alice* believes with probability-0.6 *Bob* to believe her to wear red and with probability-0.4 *Bob* to believe her to wear yellow. (**second-order belief**)
- *Alice* believes with probability-0.6 *Bob* to believe her to believe with probability-0.6 him to wear blue and with probability-0.4 him to wear green as well as with probability-0.4 *Bob* to believe her to believe him to wear green. (**third-order belief**)

Optimality Defined for Types

Definition 2

Let Γ_N be a normal form, $\mathcal{M}^\Gamma$ an epistemic model of it, $i \in I$ some player, $c_i \in C_i$ some choice of player i, and $t_i \in T_i$ some type of player i. The choice c_i is optimal for t_i, if c_i is optimal given t_i's induced conjecture.

Note: to check whether some choice is optimal for a given type,

only the first-order belief

needs to be considered – not its higher-order beliefs.

Epistemic Models and Rationality

Definition 3

Let Γ_N be a normal form, $i \in I$ some player, and $c_i \in C_i$ some choice of player i. The choice c_i is rational, if there exists an epistemic model $\mathcal{M}^\Gamma$ of Γ with a type $t_i \in T_i$ of player i such that c_i is optimal t_i.

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Iterating Belief in Rationality

- Intuitively, a choice is rational, if it is optimal for some conjecture.
- A player can then be said to believe in rationality, if he only assigns positive probability to choices & conjectures of his opponents such that the choices are optimal for the conjectures.
- Correspondingly, a player believes his opponents to believe in rationality, if he only assigns positive probability to beliefs of his opponents that believe in rationality, etc.
- In this fashion, a restriction is imposed on every layer of a player´s belief hierarchy, and this gives rise to the **epistemic condition** of **common belief in rationality**.
- Intuititively, a player expressing **common belief in rationality** thus exhibits a state of mind, where
  - he believes in rationality,
  - he believes his opponents to believe in rationality,
  - he believes his opponents to believe that their respective opponents believe in rationality,
  - etc.
- These ideas are now formalized in **epistemic models**.

Belief in Rationality

Definition 4

Let Γ_N be a normal form, $\mathcal{M}^\Gamma$ an epistemic model of it, $i \in I$ some player, and $t_i \in T_i$ some type of player i. The type t_i believes in rationality, if t_i only assigns positive probability to choice-type combinations

$$\big((c_1, t_1), \ldots, (c_{i-1}, t_{i-1}), (c_{i+1}, t_{i+1}), \ldots (c_n, t_n)\big)$$

such that c_j is optimal for t_j for all $j \in I \setminus \{i\}$.

Higher-order Beliefs in Rationality

Definition 5

Let Γ_N be a normal form, $\mathcal{M}^\Gamma$ an epistemic model of it, $i \in I$ some player, and $t_i \in T_i$ some type of player i.

- The type t_i expresses *1-fold belief in rationality*, if t_i believes in rationality.
- Let $k > 1$. The type t_i expresses *k-fold belief in rationality*, if t_i only assigns positive probability to opponents' types that express *(k-1)*-fold belief in rationality.

Let $l \geq 1$. The type t_i expresses up to *l*-fold belief in rationality, if t_i expresses *k*-fold belief in rationality for all $k \leq l$.

Common Belief in Rationality

Definition 6

Let Γ_N be a normal form, $\mathcal{M}^{\Gamma}$ an epistemic model of it, $i \in I$ some player, and $t_i \in T_i$ some type of player i. The type t_i expresses common belief in rationality, if t_i expresses k-fold belief in rationality for all $k \geq 1$.

Rational Choice under Common Belief in Rationality

Definition 7

Let Γ_N be a normal form, $i \in I$ some player, and $c_i \in C_i$ some choice of player i. The choice c_i is rational under common belief in rationality, if there exists an epistemic model $\mathcal{M}^\Gamma$ of Γ with some type $t_i \in T_i$ of player i such that

- t_i expresses common belief in rationality,
- c_i is optimal for t_i.

Illustration: An Epistemic Model for Example I

- Consider the following **epistemic model** of *Example I*.
  - Type Sets:

    $$T_{Alice} = \{t_{Alice}\}$$
    $$T_{Bob} = \{t_{Bob}\}$$

  - Beliefs for *Alice*:

    $$b_{Alice}(t_{Alice}) = (red, t_{Bob})$$

  - Beliefs for *Bob*:

    $$b_{Bob}(t_{Bob}) = (blue, t_{Alice})$$

- Observe that t_{Alice} expresses **common belief in rationality**.
  - *Alice* believes that *Bob* is of type t_{Bob} and chooses red, which is optimal for t_{Bob}.
    (*1*-fold belief in rationality)
  - *Alice* believes that *Bob* believes her to be of type t_{Alice} and to choose blue, which is optimal for t_{Alice}.
    (*2*-fold belief in rationality)
  - *Alice* believes that *Bob* believes her to believe him to be of type t_{Bob} and to choose red which is optimal for t_{Bob}.
    (*3*-fold belief in rationality)
  - etc.

Shortcut to Verifying Common Belief in Rationality

Theorem 8

Let Γ_N be a normal form and $\mathcal{M}^\Gamma$ an epistemic model of it. If all types express belief in rationality, then all types express common belief in rationality.

Proof:
(by INDUCTION on belief order k)

- INDUCTION BASE:
  - It directly holds that every type in $\mathcal{M}^\Gamma$ expresses 1-fold belief in rationality.
- INDUCTION STEP:
  - Suppose that every type expresses k^*-fold belief in rationality for some $k^* > 1$.
  - Every type thus only assigns positive probability to opponents' types that express k^*-fold belief in rationality, and consequently expresses $(k^* + 1)$-fold belief in rationality.
  - By induction, it then follows that every type expresses k-fold belief in rationality for all $k \in \mathbb{N}$.
- Therefore, every type expresses common belief in rationality.

Agenda

- Epistemic Model
- Common Belief in Rationality
- **Iterated Strict Dominance**
- Characterization of Common Belief in Rationality

Iterating Strict Dominance Arguments

- Formally, a solution concept (*SC*) in **classical game theory** is a set of choice profiles, i.e. $SC \subseteq \times_{i \in I} C_i$.
- The solution concept of **iterated strict dominance** repeatedly applies strict dominance to the game:
  - **Step 1:** *within the original game, eliminate all choices that are strictly dominated.*
  - **Step 2:** *within the reduced game obtained after* **Step 1**, *eliminate all choices that are strictly dominated.*
  - **Step 3:** *within the reduced game obtained after* **Step 2**, *eliminate all choices that are strictly dominated.*
  - *etc.*
- The solution of the game then consists of all choice profiles that can be formed by the surviving choices.

Iterated Strict Dominance

Definition 9

Let Γ_N be a normal form.

- $SD^0 := \times_{i \in I} C_i$.
- $SD^{(n+1)} := \times_{i \in I} SD_i^{(n+1)}$, where

  $SD_i^{(n+1)} := SD_i^n \setminus$

  $\{c_i \in SD_i^n : \exists r_i \in \Delta(SD_i^n)$ s.t. $U_i(c_i, c_{-i}) < V_i(r_i, c_{-i}) \forall c_{-i} \in SD_{-i}^n\}$

  for all $i \in I$ and for all $n \geq 0$.

The set $SD^k = \times_{i \in I} SD_i^k$ is called *k*-fold strict dominance for all $k > 0$, and the set $ISD := \cap_{k \geq 0} SD^k$ is called iterated strict dominance.

Illustration: ISD in Example I

- **Step 1:** $C_{Alice} = \{blue, green, red, yellow\}$ and $C_{Bob} = \{red, yellow, blue, green\}$.
  - *Alice*: yellow is strictly dominated by 0.5blue $+ 0.5$green.
  - *Bob*: green is strictly dominated by 0.5red $+ 0.5$yellow.
- **Step 2:** $SD^1_{Alice} = \{blue, green, red\}$ and $SD^1_{Bob} = \{red, yellow, blue\}$.
  - *Alice*: red is strictly dominated by green.
  - *Bob*: blue is strictly dominated by yellow.
- **Step 3:** $SD^2_{Alice} = \{blue, green\}$ and $SD^2_{Bob} = \{red, yellow\}$.
  - *Alice*: green is strictly dominated by blue.
  - *Bob*: yellow is strictly dominated by red.
- **Iterated strict dominance** yields blue for *Alice* and red for *Bob*. (Formally, $ISD = \{(blue, red)\}$.)

Example II: (*For Self-Study*) Where to Locate My Pub?

- **Step 1:** $C_{Alice} = C_{Bob} = \{a, b, c, d, e, f, g\}$.
  - *Alice* and *Bob*: *a* is strictly dominated by *b*, and *g* is strictly dominated by *f*.
- **Step 2:** $C^1_{Alice} = C^1_{Bob} = \{b, c, d, e, f\}$.
  - *Alice* and *Bob*: *b* is strictly dominated by *c*, and *f* is strictly dominated by *e*.
- **Step 2:** $C^2_{Alice} = C^2_{Bob} = \{c, d, e\}$.
  - *Alice* and *Bob*: *c* is strictly dominated by *d*, and *e* is strictly dominated by *d*.
- **Iterated strict dominance** yields *d* for *Alice* and *d* for *Bob*.

Properties of Iterated Strict Dominance

Theorem 10 (Intelligibility)

Let Γ_N be a normal form. $ISD \neq \emptyset$.

Theorem 11 (Effectiveness)

Let Γ_N be a normal form. There exists $k \in \mathbb{N}$ such that $SD^n = SD^k$ for all $n > k$.

Theorem 12 (Monotonicity)

Let Γ_N be a normal form, $i \in I$ some player, and $c_i \in C_i$ some choice of player i. If $c_i \notin SD_i^k$ for some $k \geq 0$, then c_i is strictly dominated against $SD_{-i}^{k'}$ for all $k' > k$.

Conceptual Upshots of the Three Properties

- INTELLIGIBILITY: **ISD** always returns a non-empty output and can thus be applied to any game.
- EFFECTIVENESS: **ISD** always stops after fintely many rounds and thus constitutes a finite procedure.
- MONOTONICTY: a choice identified by **ISD** as strictly dominated in some round remains strictly dominated in all succeeding rounds, and **ISD** can thus be viewed as order-independent.

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- Epistemic Model
- Common Belief in Rationality
- Iterated Strict Dominance
- **Characterization of Common Belief in Rationality**

Motivation

- The epistemic and the classical perspectives are now related to each other.
- In the Example reasoning in line with common belief in rationality and the solution concept of ISD both lead to the same result.
- As it turns out this is not a coincidence, as common belief in rationality and ISD are equivalent.

Epistemic Characterization of Iterated Strict Dominance

Theorem 13

Let Γ_N be a normal form, $i \in I$ some player, and $c_i \in C_i$ some choice of player i. The choice c_i is rational under common belief in rationality, if and only if, $c_i \in ISD_i$.

- The epistemic characterization of ISD consists of two directions.
- **Epistemic Foundation:** *CBR* implies *ISD*.
- **Existence:** ISD can be supported by *CBR*.

Intelligibility

Corollary 14

Let Γ_N be a normal form. There exists an epistemic model $\mathcal{M}^{\Gamma}$ of Γ in which all types express common belief in rationality.

- The applicability of common belief in rationality does thus not depend on any particularities of the underlying game.
- **Intelligibility** thus takes shape classically (Theorem 10) as well as epistemically (Theorem 14).

Required Background Reading for Topic 2

A. Perea (2012): *Epistemic Game Theory: Reasoning and Choice.* Cambridge University Press.

- Chapter 3 “Common belief in rationality”.